Damped mechanical oscillator: Experiment and detailed energy analysis

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Introduction

The damped oscillator is discussed in every high school textbook or introductory physics course, and a large number of papers are devoted to it in physics didactics journals. Papers typically focus on kinematic and dynamic aspects[1-5] and less often on energy. Among the latter, some are devoted to the peculiar decreasing behavior of energy characterized by ripples,[6,7] that can be easily demonstrated by using a dynamic modeling approach.[8]

In this note we consider an oscillator consisting of a cart running on a horizontal track, two springs, and a damping device created with magnets and a metal plate attached to the cart (Fig. 1). Using sensors and data acquisition software,[9] we measure kinematic quantities and three forces: those of the springs on the cart and, separately, the force between magnets and the plate. A detailed analysis of the energy exchanges between the cart and the interacting parts is obtained. In particular, we show that only the energy exchanges with the magnets are affected by dissipative processes while over a suitable time interval the net energy exchanged between cart and springs equals zero.

Experimental set up and kinematics

The oscillator is constructed from a cart with a metal plate on top and two springs. The plate interacts with two magnets attached to a force sensor (Fig. 1). Position and linear velocity are measured with a rotary motion sensor, while forces are obtained with the aid of three separated force sensors (for the interaction of springs and cart and for the interaction of cart and magnets).

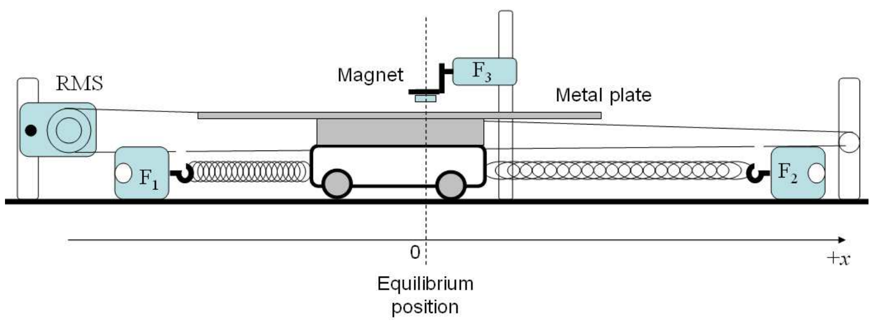
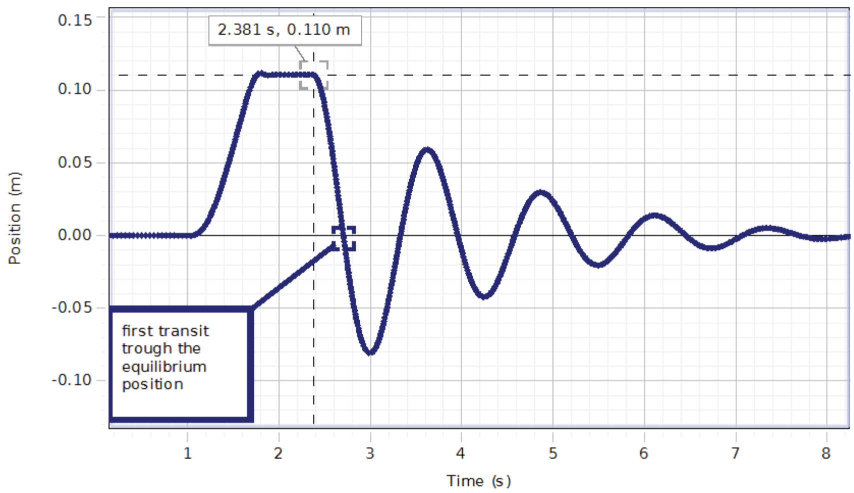


Figure 1 – Kinematic data is taken by a rotary motion sensor (RMS). The springs are fixed to two force sensors (F_1 and F_2). The magnets responsible for the dissipative interaction are attached to a third force sensor (F_3) allowing to determine the horizontal value of the damping force on the cart.

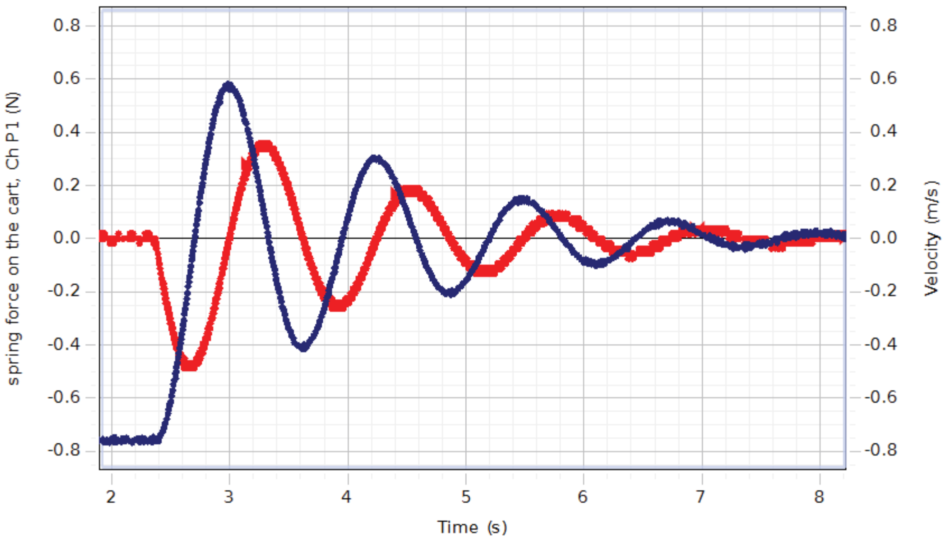
The mechanical characteristics of the oscillator (additional mass on the cart; elastic constants of the springs) can be chosen quite freely, while the strength of the magnetic interaction can be regulated via the distance between magnets and the plate. All quantities necessary for the energy analysis, i.e., for energy transfers, can be obtained from measured forces and velocity.
To perform the experiment, the cart is displaced from the equilibrium position and then released. Fig. 2 shows a typical behavior of position vs. time: the motion is damped.

Figure 2 - Cart position vs. time, showing damped oscillation. The initial position of the oscillator (0.110 m) is evidenced.



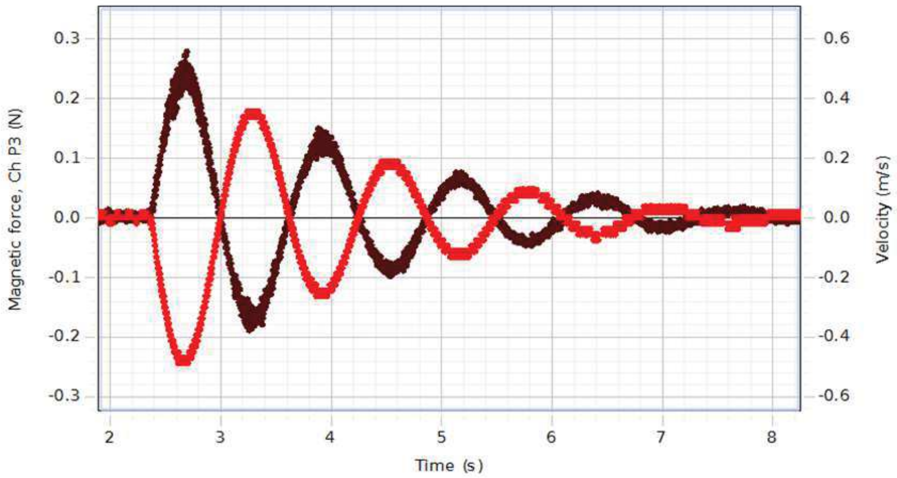
Energy
We consider now the energy aspects of the oscillator not only globally (integrated) but also by focusing on instantaneous energy exchanges between the cart and the springs and between the cart and the magnets. In order to do this, we need to find the magnitudes of the energy flows (power) that characterize the interactions. A first insight can be obtained from a graphical representation of the velocity of the cart and of the measured forces versus time (Fig. 3 and Fig. 4).

Figure 3 – Force of one spring on the cart (starting at about –0.8 N) and cart's velocity (starting at 0 m/s) vs. time: the sensor F_1 in Fig. 1 lets us measure the force of the spring (which is considered to be ideal) on the cart. The force applied by the other spring (not shown) is almost identical.



Since the velocity is zero at the turning points and has a maximum when the cart transits through the equilibrium position, we understand why measurements show a shift of a quarter of a period between the velocity and the spring force (in Fig. 3, only the velocity and the force on the cart exerted by the spring on the left, measured by sensor F1, are displayed).

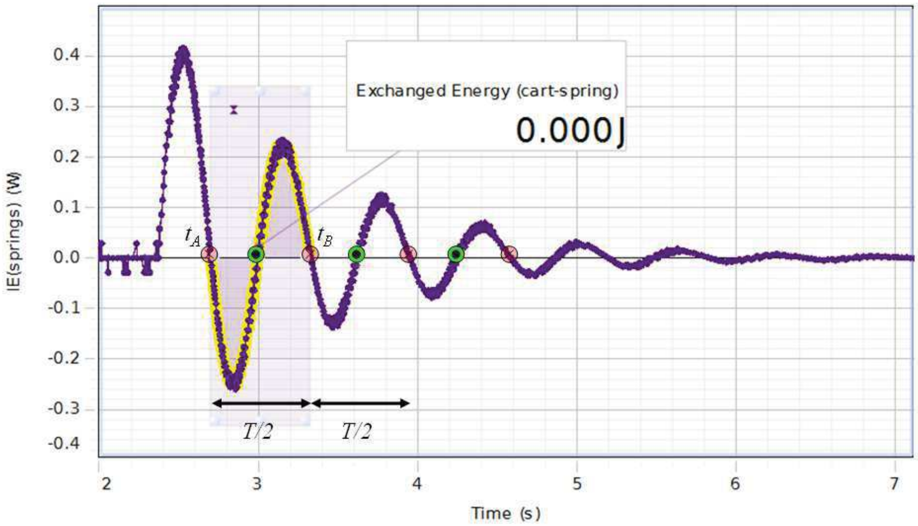
Figure 4 - Force of the magnets on the cart and the cart's velocity vs. time. The velocity becomes negative when the cart is released.



The force of the magnets on the cart is proportional to the velocity but with opposite sign (Fig. 4). This agrees with the assumption that the magnetic force is of viscous type.

We now discuss the energy flows as functions of time. At first, we consider the interaction between cart and springs shown in Fig. 5.

Figure 5 – Energy exchange rate between cart and springs: positive values mean an ingoing energy flow, i.e. the action of the springs increases the energy of the cart; a negative value means an outflow of energy from the cart, i.e. the springs receive energy from the cart.



The rate of energy transfer, i.e. the energy current I_E, is given by the instantaneous relation:

$$I_{Es}(t) = \left[F_{s1}(t) + F_{s2}(t)\right]v(t) \quad (1)$$

where we separately consider the contributions of the two springs used in the experimental set up. The energy flow is equal to zero for two states of the system: when the cart passes through the equilibrium position (the net force of the springs is zero – crossed points in Fig. 5) or, alternatively, when the cart inverts its motion (when the velocity is zero – centered points in Fig. 5). In order to understand correctly the balance of energy, it is important to coherently define the sign of an energy flow: positive for incoming, negative for outgoing. Fig. 5 is now easy to read: when the cart goes *away* from the equilibrium position, an energy transfer from the cart to the springs takes place so that the cart's energy decreases; when the cart moves back *toward* the equilibrium position, the energy flow reverses since the springs give back stored energy to the cart.

One may wonder now if it is possible to recover experimentally the simple and well known result for an ideal spring: for any properly chosen period between two consecutive passages of the cart through the equilibrium position, for instance at time t_A and t_B (crossed points in Fig. 5), the energy balance for the (ideal) springs must result in a value of zero:

(2) $$\int_{t_A}^{t_B} I_{Es}(t)\,dt = \int_{t_A}^{t_B} \left[F_{s1}(t) + F_{s2}(t)\right] v(t)\,dt = 0$$

The graphical integrating tool of the software used allows us to test this requirement: within the experimental accuracy, the value of the selected area in Fig. 5 equals zero (a correct calibration of the force sensors and an adequate choice of the sampling frequency are crucial for this result). A very important consequence of this property is that for each half period of the oscillation the time interval of the motion *away* from the equilibrium position must be shorter than that of the corresponding motion *toward* the equilibrium position, while the total time, corresponding to half a period, remains constant (Fig. 5). Only in the ideal frictionless case would the "away" and "toward" motions be of the same duration. The asymmetry of these two time intervals depends only on strength and type of the magnetic force. In Fig. 6 the absolute values of the energy exchanges in the "away" and the "toward" motion for some half periods, obtained by numerical integration, are compared.

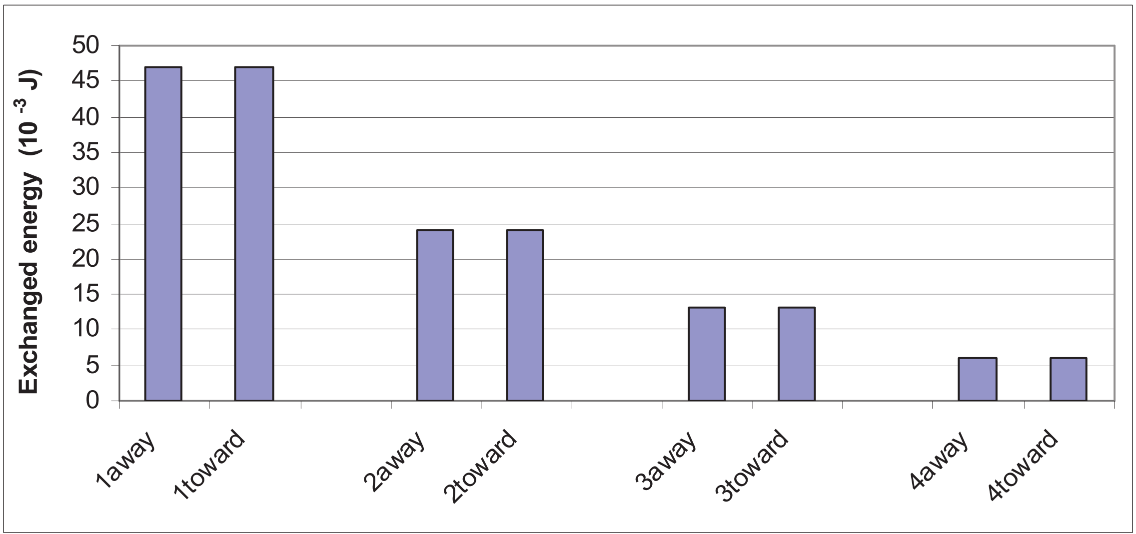


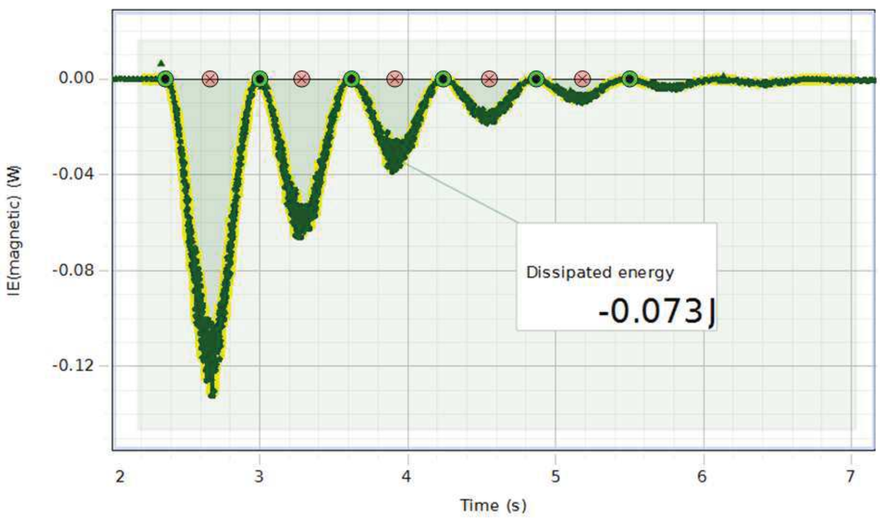
Figure 6 – Energy exchanges between two consecutive transits trough the equilibrium position: note that "away" and "toward" exchanges cancel for proper consecutive intervals while the magnitude of the exchange decreases from half-period to half-period.

In order to complete our discussion, we will now consider the energy exchange as a result of a dissipative process.
In Fig. 7, the energy flow associated with the interaction between the magnets and the plate fixed on the cart is shown. This quantity is given by the instantaneous relation:

(3) $$I_{Em}(t) = F_m(t)\ \ v(t)$$

Since the magnetic force and the velocity have opposite signs (Fig. 4), their product is always non-positive, taking the value zero only in the positions where the direction of motion is reversed and the velocity is zero (centered points, Fig. 7). Data shows that this is indeed a purely dissipative interaction.

Figure 7 – Energy exchange due to the dissipative magnetic force: the energy flow is always outgoing so that the value of the rate at which energy is exchanged is always non-positive (zero at the turning points). The integral over the whole process gives the total amount of energy dissipated in the interaction between the magnets and the plate.



One may wonder again if it is possible to recover another simple result: the energy dissipated should be given by the value of the energy initially stored in the springs:

$$(4) \qquad \int_0^\infty \left| I_{Em}(t) \right| dt \stackrel{?}{=} \frac{1}{2} K_{syst}\, A_{init}^2$$

With the help of the graphical integrating tool, it is possible to determine the value of the energy leaving the system, i.e., the energy dissipated. The resulting amount can be compared to the energy initially stored in the springs which is calculated from the measured initial position (Fig. 2). Concrete results show a difference that depends upon the strength of the magnetic damping. This result indicates that there are other sources of dissipation such as air, wheels, etc. In the example shown here, with relatively low damping, elastic spring constants of 6.8 N/m and 7.0 N/m respectively, and an initial amplitude of 11.0 cm, we obtain 0.084 J for the total initial energy stored in the springs, whereas the amount of dissipated energy equals 0.073 J.

Summary

In this note, we limited ourselves to a careful experiment and the presentation of the main features of the different energy flows in a damped mechanical oscillator. We were able to show how an experiment can clarify the differing roles played by the spring forces and the dissipative force(s) in the exchange of energy.

This study can be extended both experimentally and with the help of dynamical computer modeling. By performing additional experiments without magnetic damping, we can recognize the role of different forms of mechanical interaction in our data. A dynamical model, on the other hand, helps with a more complete quantitative analysis that will allow us to distinguish between different forms of damping (sliding and/or viscous). This combined experimental and computer modeling approach will be discussed in a forthcoming paper.

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