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# Smooth trajectory generation for industrial robots performing high precision assembly processes



Anna Valente\*, Stefano Baraldo, Emanuele Carpanzano (2)

SUPSI, Institute of Systems and Technologies for Sustainable Production, Galleria 2, Manno 6928, Switzerland

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Keywords: Robot Motion Trajectory planning ABSTRACT

Industrial robots conceived for high-precision assembly processes are demanded to match the best trade-off between precision and speed. This research presents a new approach for defining the motion profiles of robots, based on a smooth trajectory generation model. Execution time is minimized by a novel multi-variable optimization approach, taking into account the performance of each joint and the requirements of extremely precise assembly tasks. The proposed method, tested on a modular robot for the optoelectronics industry, provides jerk-bounded trajectories up to 39% faster compared to the best performing motion planning approaches, while offering the possibility to adapt these trajectories for degraded operating conditions.

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# 1. Introduction and paper motivation

Industrial robots are demanded to target extremely challenging precision and reliability performance with agile and efficient architectures. In the assembly industry of high precision, the trade-off between repeatability and speed represents a key competitive leverage for robots. Targeting cutting edge precision while being versatile (at hardware and software levels) implies researching on topics ranging from the design of more solid mechatronic robot modules with close to zero backlash [1], the design of modular and scalable kinematic models which can match production evolution with reconfiguration [2], trajectory planning and path planning which can diversify the strategy to distribute the motion across the robotic chain [3], and finally the field of modular control architecture that can detect and match changes of the robot morphology along with performing a persistent monitoring of its health and behavior [4].

The present work investigates the topic of trajectory planning for industrial robots that require a very high level of motion smoothness while executing highly precise assembly tasks. The subject of trajectory planning is traditionally addressed from the temporal perspective [5,6], the motion smoothness perspective [7] or a mix of both aspects [8]. In the field of precision manufacturing, motion smoothness is a primary aspect. To this aim, many works rely upon the use of spline functions, optimized or constrained with various approaches [9,10]. Moreover, multiple-axis movements are often managed in the operations workspace [11], although the definition of trajectories in joint space can be, in some cases, a viable simplification that allows to implement smoother

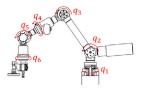
joint trajectories. In both cases, various authors proposed to achieve a higher degree of regularity by increasing the polynomial degree of the path or of the motion profile [12,13]. A non-polynomial family of smooth profiles in the joints space has been used in Ref. [14], in a movement synchronization strategy where all the joints phase their acceleration ramps on the slowest one, in a rather conservative way. A further limit for the robot motion capability is represented by the difficulty to adapt the motion planning strategies on the fly when a robot anomaly is detected at servo drives or CNC levels. As a result of that, the degrading phenomena of robots cannot be minimized or temporarily handled with a regenerative motion strategy that is optimized over the time.

The current work is motivated by the realization of ReRob I (Fig. 1), a serial robot characterized by 5  $\mu$ m repeatability designed at SUPSI University [4], whose high-precision target level raises the necessity for smooth motion planning. ReRob I presents a high level of modularity in the mechanical structure, along with a decentralized control system architecture. By relying upon a set of Key Performance Indicators (KPIs), such control architecture allows to adapt the kinematic limits of each joint and in general the trajectory of the robot, depending on the detected performance status of the robotic modules.

In this work, the authors propose a multi-variable time optimization approach for motion planning, based on smooth jerk motion profiles. These are optimized taking into account both the specific robot task and the exact allocation of the specific joint models across the robotic chain, as described in Section 2.1. Fig. 1 shows an overview of ReRob I's kinematic adaptation scheme.

The rest of the paper is organized as it follows: Section 2 outlines the approach and the mathematical formulation; Section 3 describes the test setting and compares the proposed method with other state-of-the-art approaches; Section 4 summarizes the benefits of the approach and the future steps.

<sup>\*</sup> Corresponding author. E-mail address: anna.valente@supsi.ch (A. Valente).



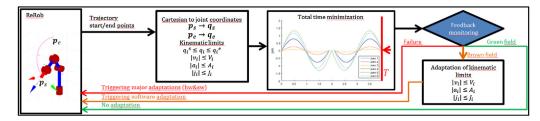


Fig. 1. Drawing of ReRob I configured as an anthropomorphic, spherical wrist manipulator (left panel) and sketch of ReRob I's trajectory planning and adaptation scheme (right panel).

#### 2. Trajectory planning

The model detailed in the current section is based on the optimization of execution time by generating trajectories in the joint space, under range constraints for acceleration, velocity and jerk. Given the starting and ending coordinates of a movement, the method is independent from the underlying kinematic model, as it works only on the motion laws for joint displacements. The chosen type of parametric motion profile is the *sine-jerk* suggested in Ref. [15], applied to the motion of multiple joints. This parametric family allows to produce smooth motion profiles, while offering an easy way to control the maximum values of jerk, acceleration and velocity.

Sine-jerk motion profiles have already been applied to multiple joints movements in Ref. [14]. Our work presents a different approach, based on the simultaneous optimization of execution time on all joints, which overcomes some limitations of the existing one, thus enabling new features:

- In Ref. [14], all the trajectory phases (acceleration, constant velocity and deceleration) are synchronized across the joints, i.e. they begin and end at the same time. Our approach allows asynchronous accelerations and decelerations, and constrains only the total movement time. This largely extends the set of admissible solutions, possibly including faster ones.
- The method in Ref. [14] determines the acceleration/deceleration
  and constant velocity times for all the joints firstly by evaluating
  singularly their fastest acceleration ramp times and constant
  velocity times, then equalizing all of them to the largest values
  found. This conservative criterion is quite far from optimality.
  Our approach optimizes numerically the execution time by
  adapting acceleration times in the best possible way.

The trajectory planning module introduced in this work relies on the following assumptions:

- The considered robot manipulator performs mainly pick-andplace tasks associated to assembly processes where very smooth pick and place movements are required, to provide an accurate positioning of the gripped component.
- Rapid movements are performed between pre-pick and preplace positions that do not require specific paths (e.g. linear) in the Cartesian workspace or avoidance of any obstacles. This implies that rapid point-to-point movements can be performed by trajectories designed in the joints space. A collision avoidance strategy, which is out of the scope of this work, could be introduced for example by iteratively adding intermediate waypoints to the trajectory.
- Though inspired by a specific configuration of ReRob I, i.e. a 6-degrees of freedom (DoF) anthropomorphic manipulator with spherical wrist, the proposed motion planning method has general purpose, and it can be adapted to any serial or parallel manipulator with arbitrary DoFs.

## 2.1. Model rationale for high precision manipulator

The industrial manipulator structure we refer to is constituted by a number of modular joints and links which form the kinematic chain. Each joint  $i = 1, \ldots, n$ , where the index i represents the serial position in the kinematic chain, is characterized by a specific joint type  $k_i$ . In particular, a typical anthropomorphic, spherical-wrist

configuration can be assembled by using 2 different types of joints: a higher torque model for the first 3 axes and a smaller, lower torque model for the 3 wrist axes, yielding  $k_1$ ,  $k_2$ ,  $k_3$  = 1 and  $k_4$ ,  $k_5$ ,  $k_6$  = 2. Each joint has different velocity, acceleration and jerk ranges, depending on the type of joint and on its position in the kinematic chain.

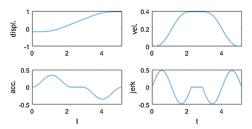
Also the specific type of assembly task and the starting and ending positions may influence the kinematic parameters for each joint: for example, movements to be performed with the empty gripper may be realized at higher speeds and accelerations with respect to movements performed while carrying a component; moreover, long-reach trajectories, which are more stressful for the actuators, may require slower movements to keep oscillations within the required specifications. The task index  $m = 1, \ldots, n_t$ allows then to differentiate control parameters between different segments of the work process. As mentioned before, this index depends on starting and ending Cartesian positions  $\mathbf{p}_s$ ,  $\mathbf{p}_e$  and task type h (e.g. pick, place, change tool, etc.), but for brevity the full dependence  $m = m(\mathbf{p}_s, \mathbf{p}_e, h)$  will be omitted in the following. The task index influences the values of motion constraints (maximum jerk  $J^{MAX}$ , maximum acceleration  $A^{MAX}$  and maximum velocity  $V^{\text{MAX}}$ , as well as path destinations), while acceleration ramps will be optimized for each task.

#### 2.2. Sine-jerk motion profile mathematical formulation

Positions  $\mathbf{p}_s$  and  $\mathbf{p}_e$  must be translated into a starting and ending position for each joint, using the proper kinematic inversion, to allow the generation of motion profiles for the joints. The algorithm we present in Section 2.3 generates trajectories in the joints space, providing motion profiles that are monotonic for each single joint. To generate joint motion profiles, we choose the expression of jerk as a function of time suggested in Ref. [15]. The proposed profile for a single motor, exemplified in Fig. 2, is

$$j(t) = \begin{cases} J\sin\left(\frac{2\pi}{\tau}t\right) & \text{for } t \in [0,\tau) \\ 0 & \text{for } t \in [\tau,\tau+T_V) \\ -J\sin\left(\frac{2\pi}{\tau}t\right) & \text{for } t \in [\tau+T_V,2\tau+T_V], \end{cases}$$
 (1)

where J is the jerk peak value,  $\tau$  is the acceleration time and  $T_V$  is the constant velocity time.



**Fig. 2.** Jerk, acceleration, velocity and displacement derived from the sinusoidal jerk model.

Notice that  $2\pi/\tau$  represents the sine-wave angular frequency. This profile exhibits very high regularity, nonetheless it allows to parameterize the displacement, velocity and acceleration profiles by few, easily interpretable parameters, and to define their values by means of a proper optimization method.

### 2.3. Joint optimization of motion profiles

Let us define D as the total required (angular) displacement for a single joint. By integration of the jerk profile described by (1) and some algebraic manipulation, it is possible to obtain the following expressions of the key motion parameters to control (see Ref. [15] setting  $J = A\pi/\tau$ ):

Peak acceleration 
$$A = \frac{J\tau}{\pi}$$
 (2)

Peak velocity 
$$V = \frac{J\tau^2}{2\pi}$$
 (3)  
Constant velocity time  $T_V = D\frac{2\pi}{J\tau^2} - \tau$ .

Constant velocity time 
$$T_V = D \frac{2\pi}{I\tau^2} - \tau$$
. (4)

To perform the smoothest possible trajectory and to avoid inducing unnecessary stresses or vibrations to the robot arm, we synchronize the motion profiles of all joints, so that they start and end movements at the same time. At the same time, we require the key quantities (jerk, acceleration, velocity) to remain under fixed values. To this end, we propose to set up a nonlinear constrained optimization problem, which takes into account all the n joints' motion profiles at once and, concurrently, leverages the motion capabilities on the relative joint position in the kinematic chain (indexed as  $i = 1, \ldots, n$  in the following), as well as the specific task requirements (indexed as  $j = 1, \ldots, n_t$ ).

We re-parameterize the quantities of interest as functions of the total move time  $T = 2\tau + T_V$  and of the *n* acceleration times  $\boldsymbol{\tau} = (\tau_1, \dots, \tau_n)$ . Since the total move is composed by the constant velocity phase and the symmetrical acceleration and deceleration phases, we have

$$T_{Vi} = T - 2\tau_i. \tag{5}$$

for  $i = 1, \ldots, n$ . Comparing (4) and (5), we can express the jerk as a

function of 
$$T$$
 and  $\tau_i$ :
$$D_i \frac{2\pi}{J\tau_i^2} - \tau_i = T - 2\tau_i \Rightarrow J(T, \tau_i) = \frac{2\pi D_i}{\tau_i^2 (T - \tau_i)}.$$
(6)

Substituting this new expression of jerk in (2)-(4) allows to set up the following optimization problem, structured with the objective function (7) that minimizes the execution time under the constraints of positive execution (8) and acceleration times (9); accomplishment of the jerk limits (10), acceleration limits (11) and velocity limits (12); total execution time at least double compared to the acceleration time (13):

for any fixed task  $m \in \{1, ..., n_t\}$ ,

$$\min_{\mathbf{r},T} T \tag{7}$$

subject to

$$T \ge 0,$$
 (8)

$$\tau_i \ge 0,$$
(9)

$$|J(T,\tau_i)| = |\frac{2\pi D_{mi}}{\tau_i^2(T-\tau_i)}| \le J_{mi}^{\text{MAX}}, \tag{10}$$

$$|A(T,\tau_i)| = |\frac{2D_{mi}}{\tau_i(T-\tau_i)}| \le A_{mi}^{\text{MAX}}, \tag{11}$$

$$|V(T,\tau_i)| = \left|\frac{D_{mi}}{T - \tau_i}\right| \le V_{mi}^{\text{MAX}},\tag{12}$$

$$T_V(T, \tau_i) = T - 2\tau_i, \tag{13}$$

for all joints  $i = 1, \ldots, n$  (without loss of generality, maximum kinematic parameters are considered symmetric around zero).

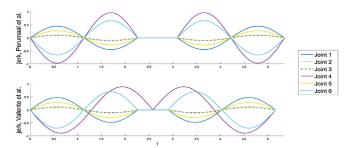


Fig. 3. Example of 6-joints jerk profiles obtained with the method described in Ref. [14] (upper panel) and the proposed method (lower panel).

Notice that the total execution time *T* is common to all the joints, while the acceleration times  $\tau_i$  are different. The beginning and ending of the movement are synchronized across the *n* joints' motion profiles, but since the synchronization is a constraint of the optimization (all the joints complete the movement at the same time T), the control parameters are automatically adapted to the best admissible performance, instead of simply rescaling all the profiles with respect to the slowest one. Fig. 3 shows a comparison between the method described in Ref. [14] and the proposed one, applied to the same  $\mathbf{p}_s$ ,  $\mathbf{p}_e$ ,  $J^{\text{MAX}}$ ,  $A^{\text{MAX}}$  and  $V^{\text{MAX}}$  data, where the asynchronous treatment of acceleration ramps allows to exploit better each joint's performances, thus yielding shorter execution times.

The target functional is linear and it is minimized under convex constraints, so any local minimizer is also a global minimizer. This optimization problem can be solved, for example, using an activeset algorithm [16].

The formulation described by (7)–(13) allows to apply customized joint management approaches, like the one suggested in Ref. [1] to perform operations in a "degraded" mode, redistributing the workload to motors that perform better (see Section 3).

### 3. Pilot case study

The proposed approach has been evaluated with regards to a number of benchmark methods [7,14,17-19], setting up ReRob's modular architecture to the proper configuration for comparison. These papers consider two benchmark tasks (point-to-point trajectories with associated kinematic limits), used also in this work to assess the effectiveness of the proposed method. The test tasks are described in Tables 1 and 2, where starting and ending positions, as well as limits for velocity, acceleration and jerk, are presented. Since the application for the proposed method is precision assembly for optoelectronics (see Fig. 4), these tasks are particularly appropriate for testing, as they reproduce typical requirements for smooth pick-and-place movements.

The results obtained generating trajectories for these tasks with the benchmark methods and the proposed one are resumed in Table 3. The method proposed in this work yields much lower execution times with respect to all the considered works. This result is achieved by pushing each joint to its best possible performance, but without violating any given constraint. For example, with regards to task 1, the resulting execution time is 39% lower than the one obtained in Ref. [14], thanks to the fact that the safe jerk range is better exploited, with respect to other methods in literature. Fig. 5 presents a comparison between a motion profile obtained on ReRob I with the algorithm presented in Ref. [14] and the one obtained with proposed method, showing the different exploitation of kinematic limits to perform faster movements.

Task 1 initial/final positions and kinematic constraints.

Joint		1	2	3	4	5	6
Position	Initial [rad]	0	$-\pi/6$	0	$-\pi/3$	0	0
	Final [rad]	$2\pi/3$	$\pi/6$	$\pi/4$	$\pi/3$	$-\pi/4$	$\pi/6$
Constraint	Velocity [rad/s]	8	10	10	5	5	5
	Acceleration [rad/s <sup>2</sup> ]	10	12	12	8	8	8
	Jerk [rad/s³]	30	40	40	20	20	20

**Table 2**Task 2 initial/final positions and kinematic constraints.

Joint		1	2	3	4	5	6
Position	Initial [°]	-10	20	15	150	30	120
	Final [°]	55	35	30	10	70	25
Constraint	Velocity [°/s]	100	95	100	150	130	110
	Acceleration [°/s2]	60	60	75	70	90	80
	Jerk [°/s³]	60	66	85	70	75	70



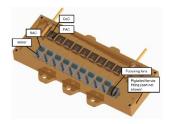
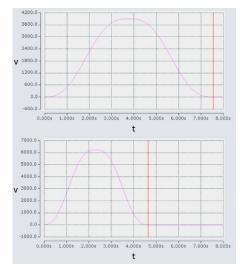


Fig. 4. Reference case: the assembly of laser multi-emitters.

**Table 3**Results of the comparison for the reference tasks.

Task	Work	Performance measure			
		Ex. time	Max % of jerk limit		
1	Gasparetto and Zanotto	9.1 s	55% (Joint 3)		
	Piazzi and Visioli	9.1 s	58% (Joint 3)		
	Simon and Isik	9.1 s	95% (Joint 3)		
	Perumaal and Natarajan	7.5398 s	23% (Joint 4)		
	Valente et al.	4.6498 s	100% (Joint 4)		
2	Chettibi et al.	2.675 s	100% (Joint 1)		
	Perumaal and Natarajan	2.5133 s	33% (Joint 4)		
	Valente et al.	1.7395 s	100% (Joint 4)		

The bold values highlight the results obtained in this work, and the fact that they achieve the best execution times.



**Fig. 5.** Velocity profiles for joint 3, task 1, obtained with Ref. [14] (upper panel) and with the proposed method (lower panel), generated on ReRob I's motion controller.

The proposed technique focuses on the specific joints, thus it enables the proper management of their degrading patterns, which results very important in precision assembly industry. Since the execution time is optimized by taking into account all the joints simultaneously, their individual performances are tuned to accommodate the overall synchronization optimally, instead of applying an unnecessary proportional, global override, which is

**Table 4** Performances with degraded joints (task 2).

Condition	Performance measure			
	Ex. time	Max jerk		
Nominal condition Degraded J4, jerk limit $20 \rightarrow 5 \text{ rad/s}^3$ Degraded J3, acc. limit $12 \rightarrow 1 \text{ rad/s}^2$ Degraded J1, vel. limit $8 \rightarrow 0.5 \text{ rad/s}$	1.7395 s 2.7613 s 2.5066 s 4.5124 s	20 rad/s <sup>3</sup> (Joint 4) 5 rad/s <sup>3</sup> (Joint 4) 6.6843 rad/s <sup>3</sup> (Joints 1, 4) 30 rad/s <sup>3</sup> (Joint 1)		

typical of traditional planning strategies. An example of this feature in action is presented in Table 4, where the sine-jerk trajectory for task 2 is compared with a jerk-degraded version (e.g. triggered by the detection of excessive vibrations) and with acceleration- and velocity-degraded versions (triggered by motor performance anomalies).

#### 4. Conclusion

The proposed method allows to set up smooth motion profiles, both in healthy and degraded status, by optimizing execution time simultaneously on all the joints of a robot manipulator. The trajectory generation method is general-purpose, and could be applied to any type of robotic kinematic chain. The kinematic limits are strictly satisfied, and they are fully exploited to achieve the fastest trajectory in the proposed smooth family. For the benchmark dataset, this approach yields execution times up to 39% shorter than the best state of the art method, while maintaining the motion profiles smooth within the given kinematic constraints. Future works will include on-line motion planning, in closed loop with the physical system sensors; moreover, the formulation could be extended to more general motion profiles, characterized by a continuous path of the robot with constrained values of velocity, acceleration and jerk, as well as with constraints on the path in the Cartesian workspace.

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