Tracking Rhythms with A Relaxation Oscillator

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Technical Report No. IDSIA-10-00

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* IDSIA was founded by the Fondazione Dalle Molle per la Qualita della Vita and is affiliated with both the Universita della Svizzera italiana (USI) and the Scuola unversitaria professionale della Svizzera italiana (SUPSI) in Lugano.

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Abstract

A number of biological and mechanical processes are typified by a continued slow accrual and fast release of energy. A nonlinear oscillator exhibiting this slow-fast behavior is called a relaxation oscillator and is used to model, for example, human heartbeat pacemaking (van der Pol & van der Mark, 1928) and neural action potential (Hodgkin & Huxley, 1952; Fitzhugh, 1961; Nagumo, Arimoto, & Yoshizawa, 1962). Similar limit cycle oscillators are used to model a wider range of behaviors including predator-prey relationships and synchrony in animal populations such as fireflies. See Winfree (1980) for a review. Though nonlinear limit-cycle oscillators have been successfully applied to beat induction (Large & Jones, 1999; Large & Kolen, 1994; McAuley, 1995; Gasser, Eck, & Port, 1999) relaxation oscillators have received less attention. In this work we offer a novel and effective relaxation oscillator model of beat induction. We outline the model in detail and provide a perturbation analysis of its response to external stimuli. In a series of simulations we expose the model to patterns from Experiment 1 of Povel and Essens (1985). We then compare the beat assignments of the model to those made by the Povel and Essens (1985) matching model. Although the overall performance of the model is very good, there are shortcomings. We believe that a network of mutually-coupled oscillators will address many of these shortcomings, and we suggest an appropriate course for future research.

1 Introduction

In this paper we show that a model of neural action potential—the Fitzhugh-Nagumo relaxation oscillator (Fitzhugh, 1961; Nagumo et al., 1962)—can form the basis of a dynamical model of beat induction. Our approach is conceptually simple: we encode a rhythm as a stream of voltage spikes and feed that voltage directly into the “axon” of the model neuron. After entrainment, we measure the phase and period alignment between neural spikes and voltage spikes in the signal. With this information we can test if the neural oscillator is spiking (“beating”) in synchrony with rhythmical downbeats. Though the oscillator does not always make appropriate downbeat predictions, it does a good job considering that the underlying mechanism is not designed to perform beat induction. For example, a single oscillator finds 34 of 35 downbeats in patterns from Povel and Essens (1985) Experiment 1 as predicted by the matching model in Povel and Essens (1985). Results from these simulations and other beat tracking simulations are presented in the current study.

1.1 Oscillator Approaches to Beat Induction

The literature on oscillators and rhythm (e.g. Gasser, Eck, and Port (1999), Miller, Scarborough, and Jones (1992), Todd, O'Boyle, and Lee (1999), Torras (1985), Dannenberg (1984), Large and Kolen (1994), Large and Jones (1999), McAuley (1995)) is significant and, due to space considerations, is not summarized here. Readers are referred to Eck (2000a) for a review. For an excellent overview of nonlinear oscillation in general, see Winfree (1980). Two particularly relevant limit-cycle oscillator beat induction devices, the Large oscillator (Large & Kolen, 1994) and the McAuley oscillator (McAuley, 1995), are briefly described below.

Large (Large & Kolen, 1994; Large & Jones, 1999) has proposed a nonlinear limit-cycle oscillator that entrains its phase to a rhythmical input by means of gradient descent. A smooth function that crosses zero at phase zero is used to establish a phase-based attractor. That function is minimized with respect to the difference between oscillator phase and the occurrence of input beats, resulting in an oscillator that continually aligns its zero phase with that of beats in the input. The Large oscillator has a second variable that modifies the slope and width of the gradient descent function such that the oscillator can sharpen its receptive field, allowing it to lock onto specific periodic components in the signal. Large and Kolen (1994) show that such an oscillator can form the basis of small connectionist networks that find salient events at multiple levels of the metrical hierarchy.

McAuley (1995, 1994) has used the term *adaptive oscillator* to describe a similar limit-cycle oscillator that entrains both its phase and its period to recurring events in a temporal signal. The McAuley oscillator entrains to a rhythmical pulse train by discretely resetting its phase to zero when a pulse is sufficiently strong. This phase resetting is governed by a cosine-shaped function that is centered around the zero phase of the oscillator. By tightening the shape of this function, the system has the ability to focus on a particular periodic component of the signal, ignoring all others. The McAuley adaptive oscillator also attempts to match its period to periodic components in the signal. This is achieved by a function that slightly slows the oscillator when phase resetting consistently occurs early and slightly accelerates the oscillator when phase resetting consistently occurs late.

Both the Large and the McAuley beat trackers are similar to the Fitzhugh-Nagumo beat tracker presented in this study. All are limit-cycle oscillators that directly use the energy in a rhythmical signal to achieve stable synchronization. The differences lie in the underlying dynamical mechanisms responsible for the stability (or relative instability) of the oscillators under different conditions. Both the Large oscillator and the McAuley oscillator use their phase adjustment functions to adjust a single phase variable, with Large using a smooth gradient descent method and McAuley using an integrate-and-fire method. The Fitzhugh-Nagumo oscillator synchronizes differently: it operates in a two-dimensional phase space with one dimension operating a different time-scale than the other. This results in limit-cycle oscillation that alternates between slow and fast movement (relating to slow uptake and fast release of voltage in a neuron).

This two-time-scale mechanism has markedly better synchronization properties than a similar single-time-scale mechanism. Namely, this slow-fast relaxation oscillation results in faster and more stable synchronization, even in large groups of mutually-coupled oscillators. Somers and Kopell (1995) offer a theory called Fast Threshold Modulation (FTM) that attributes this behavior in relaxation oscillators to the rapid modulation of firing thresholds in response to input voltage. They show that relaxation oscillators exhibit FTM but that non-relaxation oscillators in general do not, and they use this point to argue for the superiority of relaxation oscillators in tasks requiring robust synchronization. Readers are referred to Somers and Kopell (1995) for a full explanation of the theory. Because synchronization with important events of a rhythmical signal is at the heart of beat induction, and because global stability in networks of oscillators is desirable for extracting meter at multiple levels of the metrical hierarchy, we believe that a relaxation oscillator has potential as a computational element in a dynamical beat tracker.

2 Research Goals

The goal of the current study is to better understand the capacity for a relaxation oscillator to find downbeats in rhythmical signals. It is already known that relaxation oscillators are excellent synchronizers in general (Somers & Kopell, 1993, 1995). It remains an open question whether a single relaxation oscillator can make use of the hierarchically-organized structure of a rhythmical signal in order to assign downbeats. This question is addressed by exposing a relaxation oscillator to 35 patterns from Povel and Essens (1985), Experiment 1 and examining the predictions of the oscillator.

3 Model Details

Our approach to relaxation oscillator beat tracking begins with the Fitzhugh-Nagumo oscillator, derived independently by Fitzhugh (1961) and Nagumo, Arimoto, and Yoshizawa (1962) as a two-variable caricature of the four-variable Hodgkin-Huxley model of neural action potential (Hodgkin & Huxley, 1952). The Fitzhugh-Nagumo oscillator exhibits the spiking dynamics of a space-clamped neuron. When fed energy (generally from other neurons) the system gradually accrues its own voltage until it reaches a threshold; upon reaching that threshold it fires and quickly releases the energy. If the energy source is of a sufficient and constant amount, this results in stable and continuous limit-cycle oscillation.

The Fitzhugh-Nagumo relaxation oscillator is a two-variable second-order differential equation. A voltage variable (v) follows the rate of change of fast voltage release, and a voltage-recovery variable (w) follows the rate of change of slow voltage accrual. The two variables are connected to one another by a coupling connection epsilon (ϵ) such that at a certain threshold value of recovery (w), voltage (v) changes quickly. The equation for the oscillator is written as

$$\begin{aligned} \dot{v} &= -v(v-\theta)(v-1) - w + \Omega \\ \dot{w} &= \epsilon(v - \gamma w) \end{aligned} \tag{1}$$

where v is voltage, w is recovery of voltage, and Omega (Ω) is external input. Rhythms are presented to the oscillator by setting Omega (Ω) to be a time-varying signal. For all simulations in this study, this signal is a rhythmical pattern with a constant amount of voltage added to sustain oscillation. The only parameter that is varied is the coupling parameter (ϵ) that modulates the rate of flow between recovery (w) and voltage (v). This parameter is important because it controls both the period and the waveform of the oscillator: a high coupling value (ϵ) yields fast oscillation with a sinusoid-like waveform while a low coupling value (ϵ) yields slow oscillation with a highly-nonlinear waveform. This second, nonlinear oscillation is called *relaxation-mode* oscillation because the limit cycle of the oscillator closely follows or "relaxes onto" the cubic voltage isocline, as seen in Figure 1. The two remaining parameters are fixed for all simulations. They include a threshold parameter theta (θ) set at 0.2, and a shunting parameter gamma (γ) set at 1.2.

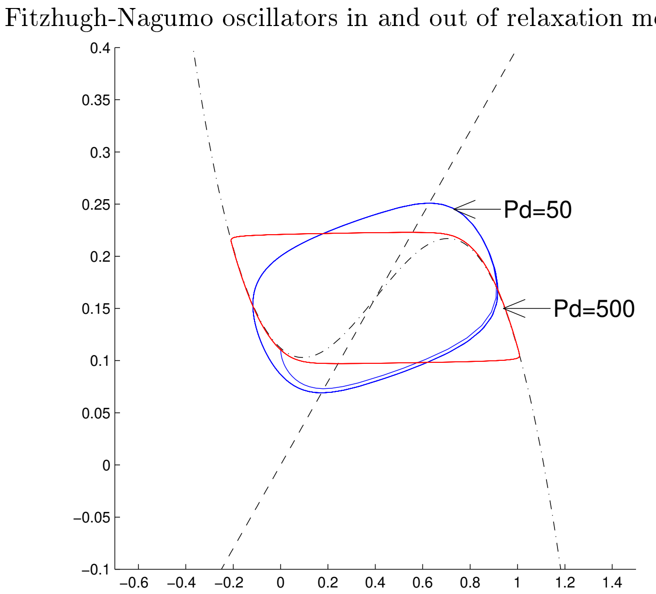

Figure 1: Two Fitzhugh-Nagumo oscillators are shown. The period 500 oscillator ($\epsilon = .0112$) has a limit cycle that is "relaxed" onto the cubic voltage isocline and so is said to be in relaxation mode. The period 50 oscillator ($\epsilon = .0289$) with a limit cycle far from the cubic voltage isocline is out of relaxation mode.

To simplify analysis, the oscillator generates a discrete output with every cycle. These beats are generated when the oscillator fires at the point in the lower part of the limit cycle where voltage (v) equals 0.4. This point is seen in the flow diagram (Figure 2) and in the time series plot (Figure 3) of the Fitzhugh-Nagumo oscillator in relaxation mode.

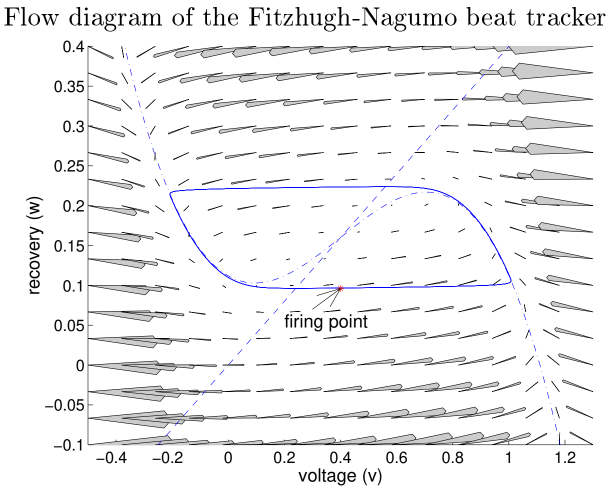

Figure 2: Observe the two distinct time-scales found in all relaxation oscillators: a slow recovery time-scale following the cubic isocline and a fast firing time-scale traveling between the two legs of the cubic isocline. The slope and magnitude of the arrows in the plot were multiplied by a factor of 10 along the recovery (w) axis so that flow in that direction can more easily be seen. Oscillator period was 500.

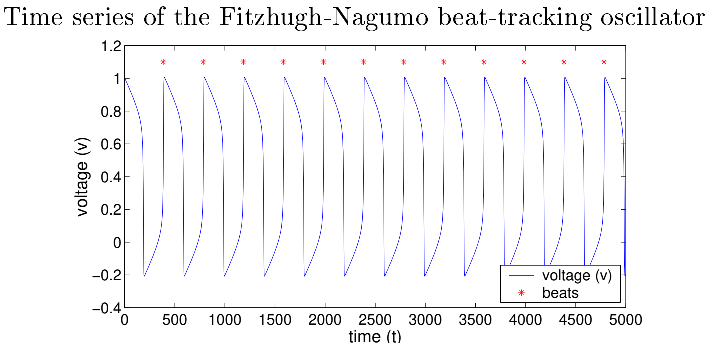

Figure 3: The time series of the Fitzhugh-Nagumo (period=500) beat tracking oscillator is shown, with discrete beats shown as stars at the top. These beats are generated so that the behavior of the oscillator can be compared with other models.

4 Perturbation Analysis

To gain a better understanding of the dynamics of the Fitzhugh-Nagumo oscillator, Figure 4 shows the results of a perturbation analysis. To generate this figure, three oscillators were instantiated at periods of 1000, 500, and 50 (ϵ = .0007, .0015 and .0293 respectively). Each oscillator was perturbed at 31 evenly-spaced points on its limit cycle. These perturbations were made by injecting extra voltage into the oscillator when it was at the desired point in its limit cycle. The y-axis of the plot shows the degree to which oscillator trajectory is changed by the perturbation. The phase values ($-\pi$ to π, on the x-axis of the plot) were generated by collapsing the (v, w) values of the limit cycle onto a single phase variable. This yields discontinuities (Izhikevich, 2000) but serves to reveal the overall dynamics of the system.

The differences between non-relaxation-mode and relaxation-mode oscillation are clear. When out of relaxation mode (period 50) the oscillator responds to input at all points in the limit cycle by following a gradient which crosses zero at Phase 0. This is akin to a simple phase pulling oscillator which continually adjusts a single phase variable when encountering input. In relaxation mode (periods 1000 and 500), the oscillator exhibits integrate-and-fire behavior. It is sensitive to input only while on the left-hand side of the cubic isocline ($-\pi$ to 0 in phase coordinates). When it does respond to input, it does so by firing. Otherwise, the oscillator clings so tightly to the cubic isocline that the perturbing voltages has little or no effect. This is akin to an integrate-and-fire oscillator with a depolarization phase. For a more detailed treatment of this analysis, see Eck (2000a).

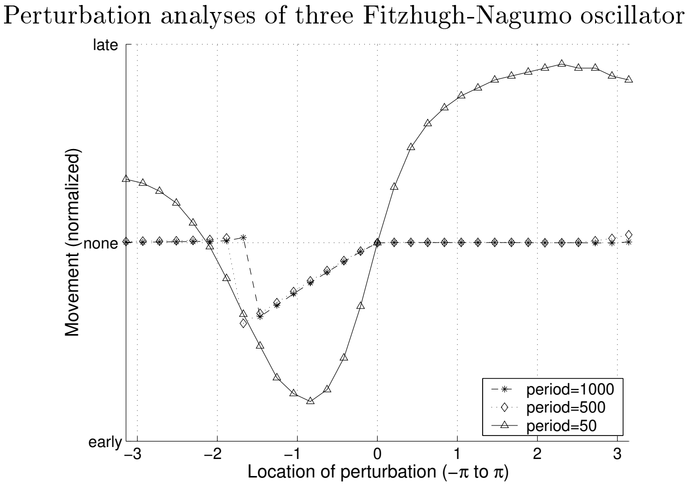

Figure 4: Perturbation analyses are shown for three Fitzhugh-Nagumo oscillators. The three oscillators have periods of 1000, 500 and 50. The x-axis shows a number of phases where an oscillator was perturbed. The y-axis shows the effect of that perturbation. Early indicates that the perturbed oscillator was advanced relative to its previous position and late indicates that the oscillator was delayed relative to its previous position. Note the qualitative difference between the perturbation responses of the oscillators in relaxation mode (periods 1000 and 500) versus out (period 50). The relaxation-mode oscillators exhibit integrate-and-fire style abrupt phase resetting while the non-relaxation-mode oscillator follow a sinusoid-shaped gradient.

5 Simulation 1—Application of Model to Patterns from Povel & Essens (1985)

5.1 The Problem

We wished to know if the oscillator model could find downbeats in rhythmical patterns. This would reveal whether the excellent synchronization properties of a relaxation oscillator do indeed extend into the domain of rhythm. Furthermore, beat induction requires of a dynamical model the crucial ability to discard irrelevant information. That is, to succeed at beat induction a model must track one particular periodic component in a rhythm (the downbeat) while ignoring others (off-beats). Success at beat induction would indicate that the Fitzhugh-Nagumo oscillator possesses this ability without need of a special input filter.

As we wished to examine the robustness of beat tracking, we ran simulations for different input strengths and different amounts of input noise. Our test set consisted of the 35 patterns from Povel

OSC values (top) Patterns (middle) & NEG values (bottom) for Povel & Essens(1985), Exp. 1							
1	*1 1 1 1* 1 1 1 1 XXXXX..XX.X.X...	2	*1 1 1 1* 1 1 1 1 XXX.X.XXX..XX...	3	*1 1 1 1* 1 1 1 1 X.XXX.XXX..XX...	4	*1 1 1 1* 1 1 1 1 X.X.XXXXX..XX...
5	*1 1 1 1* 1 1 1 1 X..XX.X.XXXXX...	6	*1 1 1 1* 1 1 1 1 XXX.XXX.XX..X...	7	*1 1 1 1* 1 1 1 1 X.XXXX.XX..XX...	8	*1 1 1 1* 1 1 1 1 XX..XXXXX.X.X...
9	*1 1 1 1* 1 1 1 1 XX..X.XXX.XXX...	10	*1 1 1 1* 1 1 1 1 X.XXX.XXXX..X...	11	*1 1 1 1* 1 1 1 1 XXX.XX..XX.XX...	12	*1 1 1 1* 1 1 1 1 XX.XXXX.X..XX...
13	*1 1 1 1* 1 1 1 1 XX.XX.XXXX..X...	14	*1 1 1 1* 1 1 1 1 XX..XX.XX.XXX...	15	*1 1 1 1* 1 1 1 1 X..XXX.XXX.XX...	16	*1 1 1 1* 1 1 1 1 XX.XXXX.XX..X...
17	*1 1 1 1* 12 12 12 12 XX.XXX.XXX..X...	18	*1 1 1 1* 12 12 12 12 XX.XXX..XX.XX...	19	*1 1 1 1* 1 1 1 1 XX..XX.XXXX.X...	20	*1 1 1 1* 12 12 12 12 XX..XX.XXX.XX...
21	*1 1 1 1* 1 1 1 1 XXXXX.XX.X..X...	22	*12 12 12 12* 1 1 1 1 XXXX.X..XXX.X...	23	*1 3 1 3 1 3 1 3* 1 3 1 3 1 3 1 3 XXX..XX.XXX.X...	24	*1 41 41 41 4* 1 41 41 41 4 X.XXX..X.XXXX...
25	*1 3 1 3 1 3 1 3* 1 1 1 1 X.X..XXXX.XXX...	26	*4 4 4 4* 4 4 4 4 XXXX.X.X..XXX...	27	*4 4 4 4* 4 4 4 4 XX.XXX.X..XXX...	28	*4 4 4 4* 1 1 1 1 XX.X..XXX.XXX...
29	*1 41 41 41 4* 1 1 1 1 X.XXXX.X..XXX...	30	*1 1 1 1* 1 1 1 1 X..XXXXX.XX.X...	31	*3 3 3 3* 3 3 3 3 XXXX.XXX..X.X...	32	*3 3 3 3* 3 3 3 3 XXXX..XX.XX.X...
33	*12 12 12 12* 12 12 12 12 XX.XXXX..XX.X...	34	*1 1 1 1* 1 1 1 1 XX.X..XXXXX.X...	35	*3 3 3 3* 3 3 3 3 X.X..XXX.XXXX...		

Table 1: Predictions from oscillator model (top) and the NEG model (middle) are shown above the patterns from Povel & Essens (1985) Experiment 1. Predictions are shown with numbers 1 through 4 indicating phase. Rests are shown with a "." and beats are show with an "X". table is ordered left to right, top to bottom.

and Essens (1985), Experiment 1. The Povel and Essens (1985) negative-evidence matching model (here called "NEG") is used to compare oscillator model predictions.

5.2 Patterns

The patterns are shown on the bottom in Table 1, with beats indicated by an "X" and rests by a "." Each of the 35 patterns consists of 16-elements: nine beats and seven rests. All patterns are permutations of the interval sequence **1 1 1 1 1 2 2 3** terminated by the interval **4**. One effect of this pattern-final interval 4 is that period-four oscillators starting at any phase other than Phase 1 are guaranteed to encounter one of the three rests at the end of the pattern. As rests generally work against the assignment of a downbeat (Cooper & Meyer, 1960; Lerdahl & Jackendoff, 1983; Povel & Essens, 1985) this accounts for the abundance of Phase 1 assignments by the oscillator model.

5.3 Method

The symbolic patterns from Povel and Essens (1985), Experiment 1 were translated into continuous temporal signals. Beats were encoded as narrow spikes. No perceptual accent rules (Povel & Okkerman, 1981) were applied, meaning that all beats had the same magnitude. Furthermore, rests were not specially marked. An additional amount (0.112) was added to all points in the signal to ensure self-sustaining oscillation. The base IOI between spikes was 125 timesteps.

Like human listeners (Garner & Gottwald, 1968) the model preferred to mark the first beat it encountered as the downbeat. To guard against this behavior, 20 oscillators were distributed high on the left-hand side of the cubic voltage (v) isocline. These oscillators were not coupled to one another, and were computed as a group only for reasons of computational efficiency. As the oscillators could only reach the limit cycle after slowly traveling down the isocline, this had the effect of evenly-distributing the oscillators on the limit cycle. In addition, the magnitude of input spikes was gradually increased during the presentation of a pattern.

To examine sensitivity to input strength, two values were used. The lower value had a mean of 0.0725 (0.0625 increasing linearly to 0.08) and the higher value had a mean of 0.0775 (0.0625 increasing linearly to 0.09).

In order to look at sensitivity to noise in the input signals, two sets of noisy signals were used in addition to the noise-free ones. Noise was injected by adjusting the inter-onset intervals (IOIs) between beats. For the low noise set, all IOIs were chosen randomly from the range -5% to +5% of the base IOI 125 (118.75 to 131.25). For the high noise set, the range was -10% to +10% (112.5 to 137.5).

Two variations in signal strength and three variations in noise yielded six separate simulations sets. All simulations were run using the ODE-solver suite in Matlab 5.3. Several solvers were tested to ensure that computational error did not affect results.

5.4 Simulation Results

Summary of Simulation Results			
Signal Strength	Signal Noise	Hits	
		Best	Both
low	none	34 (97%)	36 (88%)
low	10%	32 (91%)	33 (81%)
low	20%	29 (83%)	29 (71%)
high	none	28 (80%)	39 (95%)
high	10%	7 (20%)	7 (17%)
high	20%	8 (23%)	8 (20%)

Table 2: Summary of simulation results. The column *Best* indicates a match with the most strongly predicted downbeat assignment by the NEG model. For some patterns (17,18,20,23,24,33) the NEG model had two best downbeat assignments. In the *Both* column, all of these NEG assignments are taken into account.

Downbeat induction was measured using a binning method that compared the output beats of the oscillator to beats in the input. After allowing the system to run for 6 pattern repetitions, oscillator output beats for 2 pattern repetitions were placed into one of four bins that represented the four possible downbeat phases. If the bins did not match for both pattern repetitions, that oscillator was declared "failed". Through a large number of simulations, this was shown to be an effective measure of beat induction. Namely, using more than 6 repetitions did not yield a significant decrease in failures. Also, more than 2 binning repetitions did not yield in a significant increas in failures. This measure is a modification of a Mean Phase Coherence (MPC) method used for networks of oscillators (Somers & Kopell, 1993; Eck, 2000a).

Table 1 on page 6 shows the low-input-strength no-noise predictions from this binning method on the top along with the predictions of the NEG model in the middle. A downbeat was predicted for a given phase if the binning algorithm placed at least one oscillator in that bin. Usually, many oscillators were found in a particular bin but this was not always the case. (See Section 5.4.3 below). The model is in agreement with the best downbeat predictions of the NEG model for 34 of 35 patterns, a success rate of 97.1%. Pattern 28 was the only total failure. In two of the five cases where there were multiple correct downbeat assignments, the model found both of them (patterns 23 and 33). In three cases (patterns 22, 25 and 29) the oscillator model predicted two different downbeats while the NEG model predicted only one. Overall the performance of the model was very good. (Though the caveat is in order that we are comparing our model to another model. One direction for future research is to collect more downbeat assignment data for these and similar patterns.)

Two examples of the performance of the model are shown in figures 5 and 6. The first figure shows the oscillators tracking pattern 1. All 20 oscillators have settled on the same solution of phase 1. The second figure shows pattern 23 where the oscillators have found two plausible downbeats at phases 1 and 2.

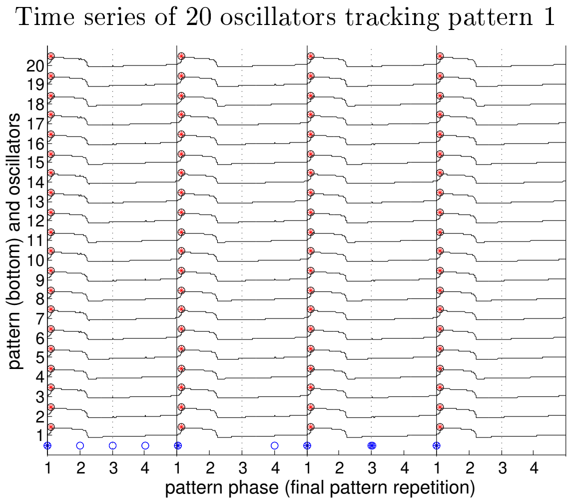

Figure 5: The time series of 20 oscillators tracking pattern 1 are shown. This plot shows only the final pattern repetition. The oscillator waveforms can be seen, with output beats marked with filled circles. The open circles at the bottom show the beats in the input. On the x-axis, the downbeat phase is shown. This example is from the no-noise low-input-strength condition. For this pattern all oscillators settled on phase 1.

Table 2 provides an overall summary of the performance data from the simulations. Effects seen in the results are discussed below.

5.4.1 The Effects of Input Signal Noise

Injecting noise into signals caused general degradation in performance but caused neither catastrophic failure nor differential behavior. In our view this shows that the oscillator is robust to noise. For example, even with 20% noise, the oscillator was able to find 29 of 35 downbeats (82.9%) with low-amplitude input signals.

5.4.2 The Effects of Input Signal Strength

Though noise did not pose a problem for the model, variations in input strength did. Namely, an increase in input strength causes the oscillator to show a marked sensitivity to "early" beats (i.e. the first beat in a series of connected beats, as well as isolated beats). In Figure 7, pattern 15 is used as an example to show this behavior. With low signal strength (top) all oscillators predict a Phase 1 downbeat, the correct prediction with respect to the NEG model. With high signal strength (bottom), however, 13 of 20 oscillators move to the incorrect Phase 4 solution, with only three oscillators settled on the Phase 1. This is an example of early-beat sensitivity: while only one beat (the first) is early in the Phase 1 solution, the Phase 4 solution has three early beats.

This sensitivity to early events can be explained in terms of integrate-and-fire dynamics: when signal strength is very strong, the first beat in a series will cause the oscillator to fire. During recovery, the oscillator is unable to respond to subsequent (perhaps more rhythmically important)

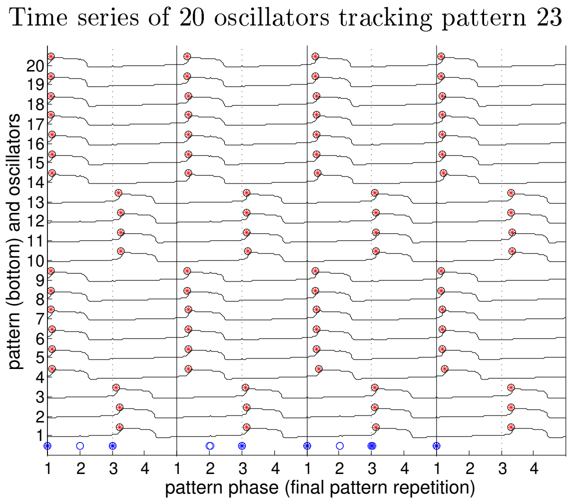

Figure 6: The time series of 20 oscillators tracking pattern 23 are shown. This example is from the no-noise low-input-strength condition. Here, the oscillators have found two solutions, phase 1 and phase 3.

beats. Recall the perturbation analysis in Section 4. Evidence of this behavior also shows up in the statistics found in Table 2 where it can be seen that success rate drops significantly for the higher amplitude, especially when noise is added to the signal.

5.4.3 Pattern Complexity

As was already mentioned, those oscillators which did not settle into periodic oscillation were treated as failures by the binning algorithm. The rate of failure was not the same for all patterns. In fact, it varied considerably. In Figure 8, these failures are plotted. As can be seen in the figure, the failure rates jump considerably at Pattern 21, which is the point in the pattern set where it becomes necessary for the model to deal with rests. (Patterns 1 through 20 are structured such that at least one period-4 downbeat solution has no rests at all). Two different NEG complexity ratings are also plotted, one with weight $W = 4$ and another with weight $W = 10$. The behavior of the oscillator model is matched very well by the $W = 10$. With this weight setting, rests have a very strong negative influence on the complexity of the pattern. For more on the use of matching models to generate complexity ratings, see (Eck, 2000b).

This drastic failure for patterns 21 through 35 can be attributed to integrate-and-fire dynamics. When a rest is encountered in the place of a beat, the oscillator may not receive sufficient energy to fire. This can cause the oscillator to fall behind slightly, rendering it susceptible to firing when the next beat occurs. Though falling behind is a problem for any oscillator, it is particularly severe for a relaxation oscillator because of the all-or-nothing behavior of integrate-and-fire dynamics: the oscillator does not adjust its phase gradually in response to input but rather fires completely causing drastic failure when failure occurs. In this way, the same dynamical mechanism which makes the oscillator a good synchronization mechanism also makes it apt to synchronize in ways which are unintended.

6 Future Research

Though these simulations reveal good performance, the oscillator model suffers from some shortfallings. First, it is overly sensitive to variations in input amplitude, and second it suffers drastic failure when it is required to make beat assignments at a phase where there is a rest in the input. Preliminary research in networks of relaxation oscillators (Eck, 2000a) suggests that these prob-

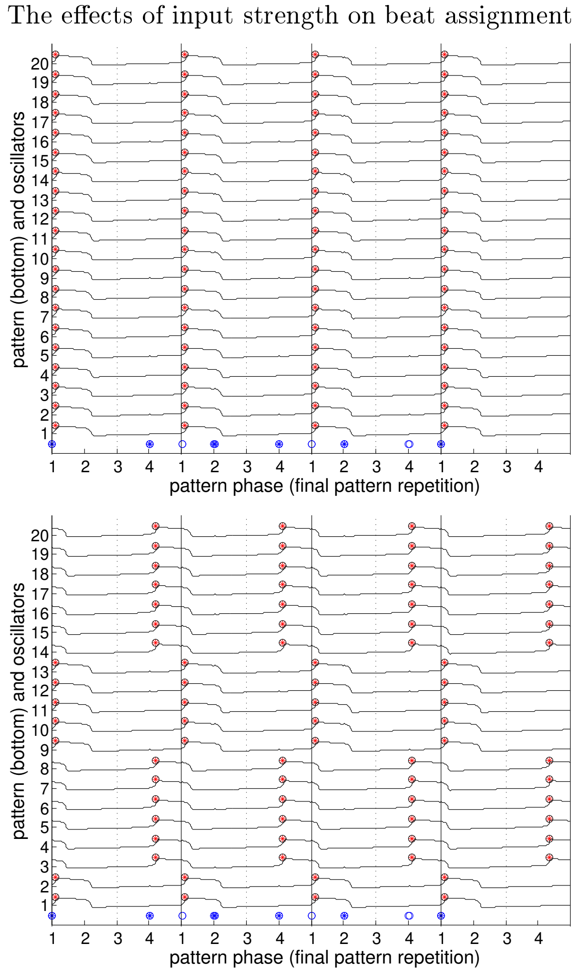

Figure 7: On the top is the pattern 15 presentation with low input strength. Here, all oscillators align with the predicted phase 1 downbeat. On the bottom is the pattern 15 presentation with high input strength. Here, some oscillators settle at phase 4, a solution not predicted by the NEG model. The high input strength causes the oscillators to be overly sensitive to early stressed beats.

lems can be overcome by taking advantage of the added stability of multiple oscillators. A model having several mutually-coupled oscillators is less sensitive to changes in input strength and is also much less likely to suffer drastic failure when rests are encountered. For example, the number of failed oscillators in the low-input-strength no-noise case was 166 of 700 (23.7%). Recall Figure 8. With a network of oscillators, similar conditions yield only 26 of 700 failures (3.7% failures).

7 Conclusions

It is clear that a single relaxation oscillator has great promise as a beat tracking mechanism. It is able to find downbeats in a set of fairly complicated rhythmical patterns, revealing that integrate-and-fire relaxation dynamics do give rise to the crucial ability to focus on one periodic component in the input while ignoring others. Though more research is needed, we believe the current study has successfully extended the findings on robust oscillator synchrony (Somers & Kopell, 1995) into the domain of rhythmical beat tracking.

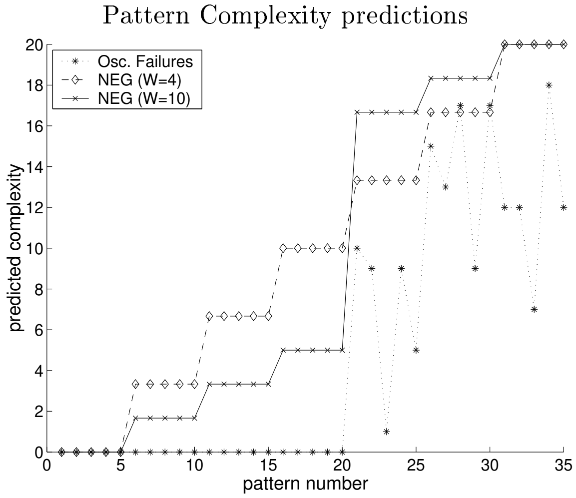

Figure 8: The number of failed oscillators for each pattern is plotted, along with normalized complexity predictions from the NEG model. Two NEG weights (W) are shown. This weight adjusts the relative negative effect of rests. The performance of the oscillator model is more in accordance with NEG when weight W is very high ($W = 10$). This indicates that the oscillator mechanism fails more often when rests are encountered.

8 Acknowledgements

Thanks to Nancy Kopell, David Somers and Art Winfree for insights on oscillator theory. This research was supported by SNF project 21-49144.96.

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