FORMATIVE ASSESSMENT IN THE TEACHING AND LEARNING OF MATHEMATICS: TEACHERS' AND STUDENTS' BELIEFS ABOUT MATHEMATICAL ERROR

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Key words: formative assessment, mathematics, errors, teachers, students, beliefs.

1. Introduction

This contribution is part of the research activities of the LLP Comenius Project FAMT&L[1], (Formative assessment for teaching and learning mathematics) which aims to promote the use of formative assessment as an element that can improve mathematics education, as it is evident for many years that the levels of success in mathematics are limited. In the theoretical orientation phase of the research project, the partner members have done a careful reflection on the training of teachers in service, based especially on some of the latest research focused on methods of training courses that use a collaborative approach in the medium-long term (Palincsar, Magnusson, Marano, Ford & Brown, 1998; Vannini, 2012). In particular, methodologies of reflective perspective that have a positive effect on teachers' ability for becoming aware of their beliefs (Cherubini, 2002) and for conducting a critical analysis of teaching practices that can be assessed as effective or ineffective (Vannini, 2012) have been adopted. To promote what is called "reflexive competence" of teachers, it was necessary to start the project by the recognition of the beliefs and daily teaching practices, according to which we can identify the training needs to be answered through the creation of training models and tools, which should promote critical reflection and the subsequent adoption of the most effective teaching methods.

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Thus, the study of teachers' conceptions of assessment is important because evidence exists that teachers' conceptions of teaching, learning, and curricula influence strongly how they teach and what students learn or achieve (Thompson, 1992). Furthermore, teachers have also a remarkable influence on students' construction of their beliefs through the ways in which they present the subject, the kinds of task they set, the assessment methods, procedures and criteria (Pehkonen, 1998). In relation to these, the main objectives of this study consist in realizing a survey on the mathematics teachers' beliefs and practices concerning formative assessment in classroom. In this contribution the discussion will be developed according to the students' and teachers' beliefs about mathematical error and the differentiation of these beliefs between different educational systems.

2. Theoretical framework

2.1. What is formative assessment?

The National Council of Teachers of Mathematics Assessment Standard (NCTM, 1995) define assessment as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition towards mathematics and of making inferences from that evidence for a variety of purposes" (p.3). In accordance to this, Harlen (2000) points out that "children have a role in assessment for this purpose since it is, after all, the children who do the learning" (p.112). That is why many researchers stress that assessment must be formed "for" learning and not "of" learning, as it is generally acknowledged that increased use of formative assessment (or assessment for learning) leads to higher quality learning (Wiliam, Lee, Harrison & Black, 2004).

In this sense, Nicol and Macfarlane-Dick (2004) argued that formative assessment should be an integral part of teaching and learning in higher education. We agree with this opinion, because of the fact that formative assessment is useful in the learning process; it explains how well the process progresses and it guides students what they wish to learn. In addition, we agree with the aforementioned opinion, emphasizing that the use of formative assessment in teaching can have many benefits on one hand on improving the students' mathematical learning but also the development of positive beliefs towards the learning of mathematics, and on the other hand in helping the teachers in doing proper adjustments according to their students' needs. Formative assessment develops fully autonomous learners, who can self-assess

their work, make meaningful inferences from it and plan the next steps for further progress (Black and Wiliam, 1998).

Formative assessment also provides information to teachers about students' difficulties and where to focus their teaching efforts. Our opinion is also in line with other researchers' definitions (e.g. Black and Wiliam, 1998) that stress the effects of formative assessment in modifying learning in relation to the students' needs. Van De Walle, Karp and Bay-Williams (2013) define formative assessment as "an along the way evaluation that monitors who is learning and who is not and helps teachers to form the next lesson". Wiliam (2007) claims also that "to be formative, assessment must include a recipe for future action" (p.41). Formative assessment then is a strategic process which uses evidence regarding the extent of student knowledge (declarative knowledge) and skill (procedural knowledge) to support further learning (Clark, 2011a) and as such increases student motivation, engagement and achievement (Cauley & McMillan, 2010).

Additionally to the aforementioned focus points about the monitoring of teaching and learning, the role of feedback is also emphasized in many other definitions about formative assessment (Sadler, 1998; Cauley & McMillan, 2010; Nicol & Macfarlane-Dick, 2004). High-quality formative assessment always puts student learning at the center thus providing feedback to individuals on specific skill areas enhance students' learning (Century Island School, February, 1999 in Lee & Wiliam, 2005).

Summarizing, a definition combining all the points stressed previously is the one provided by Popham (2008, p.5), who characterizes formative assessment as "a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes". This definition is accepted by the Formative Assessment for Teachers and Students (FAST) group as the most accessible to educators (Clark, 2011b; Melmer, Burmaster, & James, 2008).

2.2. Techniques of formative assessment

In fact, assessment practices and their outcomes on the students' learning, but also their affective domain have drawn the interest of different researchers in the last 30 years (i.e Crooks, 1988; Black & Wiliam, 1998). Previous works suggest different formative assessment techniques, most of which appear to have common points. Cauley and McMillan (2010) provide particular techniques that should

be used in teaching for the effective integration of formative assessment in instruction. Specifically, informal observations and oral questions posed to students while content is being taught or reviewed is a practice that allows ongoing formative assessment. And if the information from the observations and questions to students is accurate, the teacher identifies instructional adjustments that can help improve the students' learning.

From our experiences, observations and oral questions are included in the teachers' repertoire of formative assessment techniques and are very commonly used in teaching. Despite the frequency of their use, we are not sure that these techniques can be included among the most effective ones for formative assessment. However, research indicates that teachers' opinions and preferences are not totally in line with our opinion. In fact, Kyriakides and Campbell (1999) examined primary teachers' opinions about the appropriateness of particular techniques of assessment in mathematics. Performance test and structured observation were considered to be the most appropriate methods. On the other hand, unstructured observation and oral question-and-answer were seen to be the least appropriate techniques.

Black and Wiliam (2009) claim that high-quality formative assessment takes many forms (quality, advice and guidance, not comparison and feedback), but it always has some standards. For example, a high-quality formative assessment emphasizes the quality rather than the quantity of student work. Moreover, it targets in giving advice and guidance over giving grades, it avoids comparing students in favor of enabling individual students to assess their own learning, fosters dialogues that explore understandings rather than lectures that present information. Nevertheless, whatever the form the formative assessment takes, it should aim acquiring the knowledge of the task rather than the student (Kluger & DeNisi, 1996). The emphasis and detail provided in this type of assessment aims to give students a clear idea of what, why, and how to proceed, as they continue to work on a mission or an assignment. On the other hand, this method should be offered into manageable pieces of work so that students are not overwhelmed such processing (Clariana, 1990).

Last but not least, Brown (2004) claims that any assessment strategy that aims to be *inclusive* should establish a range of techniques for assessment (for example written assignments, presentations, reflective accounts and so on), so that the same students are not always impoverished. All participants need to be provided with *the same*

opportunities to show their abilities and potentials. This indicates that the assessment criteria need to be clear, explicit, framed in language that is worthwhile to staff and students and available well in advance of the beginning of activities that will eventually be assessed.

2.3. The use of students' errors

The use of students' errors is an important dimension of formative assessment, as it helps teachers modify their practices for helping the students correcting them, but also the students in identifying their weaknesses and try overcoming them. We strongly agree with this, thus in this section the importance of using and interpreting the students' errors is going to be discussed, emphasizing on the role of the teachers' and students' beliefs about errors, which is a part of our objectives for our project.

Wragg (2001) supports that "if students are to learn from their assessment, then correction of errors and discussion of what they have done is essential" (p.74). This strengtheners our opinion about the significance of studying teachers' beliefs regarding the origination of the students' errors, as these beliefs can affect the way the teachers will decide to discuss about errors and work with them for helping the students overcoming them.

In fact, the identification of mistakes helps teachers decide how to identify and meet pupils' learning needs and how to use their teaching time and their resources (Kyriakides, 1999). The reason on which the teachers attribute the errors will affect their decisions for their future intervention teaching practices. Therefore, the students' errors can have a formative use, as the teachers can exploit this information for modifying their future actions (Gagatsis & Kyriakides, 2000). Thus, decisions about the next learning steps follow from the formative identification of pupils' errors (Desforges, 1989). And this is particularly important, because a teaching plan which is organized in such a way, might help teachers to plan class and individual programs of work according to the different performance levels of the pupils (Gagatsis & Kyriakides, 2000).

Therefore, in order to be able to provide suggestions through our project towards the effective formative use of students' errors we have to get an insight to the teachers' beliefs about the source of these errors. A number of studies (Milhaud, 1980; Charnay, 1989; Economou, 1995) revealed that teachers attributed errors mainly to the pupils' lack of

interest or lack of preparation. Gagatsis and Christou (1997) examined also the extent to which the didactical and epistemological approaches to the concept of error influence teachers' attitudes investigated. They actually examined the interpretations that primary school teachers give about their pupils' errors. The results of their study showed that the majority of teachers hold similar beliefs. For example, 90% of primary school teachers attributed errors to the psychological situation of the pupil, 80% of the teachers attributed errors to the limited capabilities of the pupil, and 85% considered the lack of knowledge as a reason for errors.

Gagatsis and Kyriakides (2000), examined not only whether teachers agreed with aspects of the didactical and epistemological approach to the concept of error but also whether they could identify errors of their pupils associated with the concepts of obstacle and didactic contract. In their study, teachers' responses revealed that items concerned with reasons for errors can be classified into four broad categories. These are pupils' characteristics, teachers' role, the mathematical knowledge, and the rules which pupils are supposed to follow in a typical mathematics classroom:

1. The first factor is related to items which imply that errors are a negative behavior. Errors are seen as the result of "confusion" (Economou, 1995) and thereby pupils' lack of interest and/or preparation are the main reasons for errors.

2. The second factor is concerned with the role that the teacher has to play in order to enable pupils to avoid mistakes and is very significant educationally. A significant contribution of this study to educational theory on reasons associated with mathematical errors has to do with the other two factors which emerged.

3. The items associated with the third factor partly derive from the epistemological approach to the concept of error and especially with the concept of obstacle.

4. Finally, the fourth factor is highly correlated with items concerned with the concept of didactic contract.

The survey, also, showed that teachers supported that errors in mathematics are often associated with the characteristics of the pupils. This seems to be in line with the findings of a number of studies (i.e Charnay, 1989; Economou, 1995; Milhaud, 1980) which revealed that teachers attributed errors mainly to the pupils' lack of interest or lack of

preparation. Finally, the teachers considered error analysis as a significant way of improving their teaching practice.

In a more recent study, in the context of the LLP Comenius Project FAMT&L, we have presented a first comparison of Cypriot and Italian teachers and students beliefs about errors in Mathematics in the ASI9 conference on the Implicative Statistical Analysis (Christodoulou, Gagatsis, Anastasiadou, 2017). In the section 4.3 of the present text we have included a more structured presentation of the above mentioned dimension of the project FAMT&L.

2.4. Teachers' beliefs on mathematics and assessment

The study of teachers' conceptions of assessment is important because evidence exists that teachers' conceptions of teaching, learning, and curricula influence strongly how they teach and what students learn or achieve (Thompson, 1992; Calderhead, 1996). Indeed, teachers' beliefs about student self-confidence, morale, creativity, and work are "closely linked to one's choice of evaluation techniques" (Asch, 1976, p. 18).

There is a paucity of research on what teachers believe about the purpose of assessment and how they use data they collect during the assessment process, despite the fact that much has been written about the purposes of assessment. The NCTM Assessment Standards (1995) note four purposes – promoting student growth, improving instruction, recognizing accomplishments, and modifying programs. Such purposes have two foci – teachers and learners. Clarke, Clarke and Lovitt (1990) claim that the major uses of assessment focus on three areas – teachers (to improve instruction), students (to inform them on their strengths and weaknesses), and parents (so they can give support).

These three factors indicate that beliefs about the uses of assessment fall into three main categories - to inform the teacher, to inform the learners, and for accountability purposes. The factors correspond closely to the three focus areas proposed by Clarke, Clarke and Lovitt (1990). The first factor is essentially about teachers evaluating their teaching, and reflects feedback and planning components, corresponding to two of the purposes of assessment stated in the NCTM Standards (NCTM, 1995), namely, improving instruction and modifying programs. The second factor supports the notion of assessment promoting student growth, recognizing accomplishments, and giving feedback on students' strengths and weaknesses.

Many teachers are content with conservative methods of assessment because they know that good results from rote teaching enhance their image. Consequently, teachers are not pioneers in alternative strategies of assessment. They believe that their use of new models of assessment would invite educational authorities, parents and school heads to negatively assess the quality of their teaching (Gao, Du & Yu, 2006). These factors contribute to teacher's complex and contradictory conceptions of assessment. Brown (2004) further argues that the various conceptions might interact with each other and that these conceptions can lead to different practices, which are often in tension with the original purposes.

All pedagogical acts, including teachers' perceptions and evaluations of student behavior and performance (i.e., assessment), are affected by the conceptions teachers have about many educational artifacts, such as teaching, learning, assessment, curriculum, and teacher efficacy. It is critical that such conceptions and the relationships of those conceptions among and between each other are made explicit and visible.

2.5. Students' beliefs on mathematics and assessment

Students' general beliefs about the nature and acquisition of knowledge, namely epistemological beliefs, have been investigated regarding their influence on text comprehension and meta-comprehension (Kardash & Howell, 2000), problem solving (Schraw, Dunkle, & Bendixen, 1995), and conceptual change (Mason, 2000). Students' beliefs have been investigated not only as general convictions, but also as convictions about knowing and learning in specific domains, including mathematics (De Corte & Op't Eynde, 2002).

Schoenfeld (1983) pointed out the existence of a system of beliefs that drives students' behavior when trying to solve mathematical problems, since problem-solving performance cannot be seen as purely cognitive. He revealed that students' beliefs about what is useful in learning math affects the cognitive resources available to them when learning in this domain, making a large portion of stored information inaccessible when the beliefs impede rather than facilitate understanding. Garofalo (1989) pointed out different kinds of students' beliefs affecting mathematical performance, for example: the difficulty of math's problem is due to the size and quantity of the numbers, all problems can be solved by performing one arithmetical operation, in

rare cases two, the operation to be performed is determined by the keywords of the problem, usually introduced in the last sentence or in the question, thus it is not necessary to read the whole text of the problem and the decision to check what has been done depends on how much time is available.

Given that teachers' beliefs, as reflected in their practice, influence students' beliefs, it appears evident that pre- and in-service teacher training should include activities aimed at making them manifest, and encourage teachers to analyze and reflect on their own convictions about the discipline and different ways in which it can be approached in the classroom (Franke, Fennema, & Carpenter, 1997). Furthermore, students' conceptions of assessment are of particular importance because assessment has a significant impact on the quality of learning (Ramsden 1997).

The research literature on students' conceptions of assessment is not vast, and is largely focused on tertiary or higher education students (Michael-Chrysanthou & Gagatsis, 2015; Struyven, Dochy & Janssens, 2005). Review of the empirical literature on students' conceptions of the purposes of assessment has identified four major purposes, some of which can be matched to teachers' conceptions of assessment. Students are reported as conceiving of assessment as (a) improving achievement, (b) a means for making them accountable, (c) being irrelevant, and (d) being enjoyable.

3. The FAMT&L project

The FAMT&L project proposes an innovative path that, starting from an investigation of the mathematics teachers' beliefs about formative assessment, will get to design a virtual environment (a web repository) for in-service teachers' training. This learning environment should provide a variety of tools and objects (examples of learning contexts, video of situations of teaching mathematics, assessment tools, training paths and their specific use in the teaching of mathematics), including a guideline to be used in in-service secondary schools teachers training courses (more details about the program can be found in Michael-Chrysanthou, Gagatsis & Vannini, 2014).

There are five EU partners in the project: The Alma Mater Studiorum Università di Bologna – Departments of Education and Mathematics, which is the Project Coordinator, the University of Cyprus

– Department of Education, the University of Applied Sciences and Arts of Southern Switzerland – Department of Formation and Learning, the Cergy-Pontoise University – University Institute of Teachers Training and the Inholland University of Applied Sciences.

The main objectives of this project consist in realizing a survey on the mathematics teachers' beliefs and practices concerning assessment in classroom, in designing and implementing a web repository for the mathematics teachers training about the proper use of formative assessment in teaching-learning situations and in elaborating a training model (or methodology) for mathematics teachers training in secondary school. This training methodology should improve teachers' skills regarding the use of formative assessment in mathematics education in order to promote effective learning for all students.

The first part of the project is dedicated to the analysis of teachers' learning beliefs and needs regarding formative assessment, through specific qualitative and quantitative research methods (observations, interviews, questionnaires, survey, etc.). The collected data will be used for designing an effective training model for the teachers of mathematics. The design and development of this training model will be realized as an action-training research, where teachers will be actively involved and trained to develop mathematics teaching and assessing competences as well as transversal competences such as reflexive practice, self-assessment, planning and reporting methods, professional empowerment. All the products will be included in the web repository in order to give to trainees as much stimulus and tools as possible.

In this paper we are going to describe and discuss the study of mathematics teachers' and students' beliefs for formative assessment and especially the role of mathematical error. For this purpose, a questionnaire was developed, for collecting the teachers' beliefs regarding various dimensions of formative assessment. The procedure for developing our research tool is described in the next session.

3.1. The adopted definition of the project

Based on the results of the literature review, a synthesis of different definitions was done in order to be able to express the way formative assessment in mathematics teaching and learning is defined in our project.

Therefore, according to our synthesis, we resulted in providing the following extended definition and description of formative assessment.

"Formative assessment is connected with a concept of learning, according to which all students are able to acquire, at an adequate level, the basic skills of a discipline. The learning passes through the use of teaching methodologies which can respond effectively to different learning times for each student, their different learning styles, and their zones of proximal development. Formative assessment is an assessment FOR teaching and learning. It is part of the teaching-learning process and regulates it. It identifies, in an analytical way, the strengths and weaknesses of student's learning, in order to allow teachers to reflect on and modify their own practices. It allows, in a form of formative feedback, to establish a dialogue between teacher and student and to design educational interventions; It also promotes and fosters the learning of all students through differentiated teaching that ensures each student different rhythms and different teaching and learning strategies, involving at the same time the student in the analysis of own errors/weaknesses and own ability to promote self-assessment and peer-assessment and active participation in the teaching-learning process.

It is intended to give information, feedback and feed forward – in and outside of the classroom – related to the development of mathematical life-skills. In particular, it involves the different components of mathematical learning of the students (conceptual, procedural, semiotic, communicative, problem posing and solving aspects, misconceptions, organization of mathematical experience), the students' beliefs, the students' image of mathematics and of specific segments of mathematics, their behavior and classroom interaction when involved in different mathematical tasks and the outputs of teacher's choices (transposition of mathematical contents, interface between contents and methods)".

4. Research on teachers' and students' beliefs about errors in Mathematics

4.1. Methodology

In this contribution we present a part of the survey in the context of the European Program FAMT&L, which focuses on the study of beliefs about the use of mathematical errors in formative assessment. A questionnaire focused on these beliefs was administrated to all

participants (see Table 1 and Table 2 in Appendix). Besides tracing the teachers' and students' beliefs about the specific dimension of FA in each country, we proceeded to comparisons between the different countries. Further on, we are going to present the results of two such comparisons.

1. At first, a comparison between Cypriot and Swiss schools will be presented. In specific, 308 Cypriot students and 340 Swiss students participated in the present study. Respectively, 65 Cypriot teachers and 69 Swiss teachers filled also a questionnaire.

2. Then, the Cypriots' teachers' and students' beliefs will be compared to the Italians' teachers' and students' beliefs. Particularly, 308 Cypriot students and 460 Italian students participated in the present study. Respectively, 65 Cypriot teachers and 58 Italian teachers filled the questionnaire.

The analysis of the data collected in this study were conducted using the computer software Classification Hiérarchique, Implicative et Cohésitive (C.H.I.C.) (Gras, Suzuki, Guillet & Spagnolo, 2008), which gives the similarities diagrams. The similarity groups appear in an ascending manner as a function of their strength. Thus, the similarity groups are represented in a hierarchically constructed similarity diagram, which allows us to study and interpret groups of items based on resemblance of performance characteristics. This analysis aims to answer the following two questions:

(a) "What are the students' and teachers' beliefs about mathematical error in each country (Cyprus, Italy, Switzerland)?" and

(b) "How teachers' and students' beliefs about the mathematical errors differentiate between the three countries under study?"

4.2. A comparison between Cypriot and Swiss teachers' and students' beliefs about errors in Mathematics

4.2.1. The concept of the mathematical error in Cypriot and Swiss students

The Similarity Diagrams below (Figure 1) presents the Cypriot and Swiss students' beliefs about the concept of the mathematical error separately.

At a first glance the first diagram we observe that two similarity groups, which are not linked, are formed. This is an indication that the Cypriot secondary school students don't have an overview of the concept of error in mathematics, but they isolate some of its aspects. However, in the second diagram seems that Swiss students have an overview of the concept of mathematical error.

In more details, we observe that the two similarity diagrams are almost identical. In specific, we notice that strongest similarity pair is formed by the variables R4 and R5, which appear significant similarity index in both diagrams. These variables are lying in the first similarity group and they are related with the teacher's role in the handling of the student's errors in mathematics. Strong similarity index is also observed between the variables R1 and R3, which refer to the understanding of the mathematical concept, which gained through the correction of errors.

The above two pairs of variables compose the first similarity group of the diagram presenting the Cypriots' students beliefs about mathematical error and it is formed by variables related to the students' cognitive domain and present significant similarity index between them. Another different between the two diagrams appears in the variables R2 and R6, which refer to the affective domain of the students. This pair of variables forms the second similarity group in the Cypriot diagram and it presents similarity index equal to 0.63. In contrast, the variables R2 and are R6 are not linked in the diagram presenting the Swiss students' beliefs. This is an indication that Swiss students handle the errors in mathematics taking into account both their cognitive and affective domain. The variable R6 does not associated with any of the other variables in the similarity diagram. This is due to the fact that this variable refers explicitly to the "grade" in mathematics ("If I make mistakes in math, I deserve a low grade"). In contrast, the other variables mainly refer to the formative assessment in mathematics and they are not mentioned in grade.

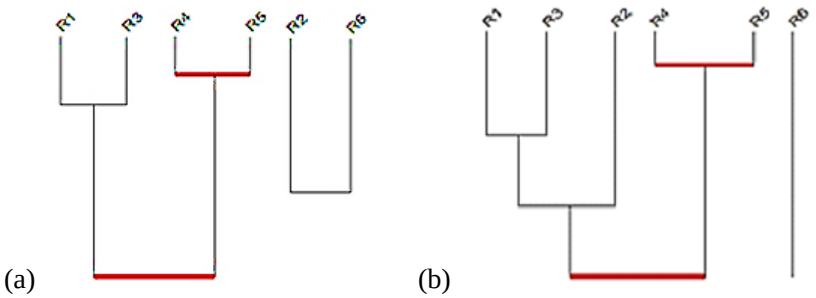

Figure 1: (a) Similarity diagram on the Cypriot students' beliefs about errors in Mathematics, and (b) Similarity diagram on the Swiss students' beliefs about errors in Mathematics

4.2.2. The concept of the mathematical error in Cypriot and Swiss teachers

The following similarity diagram (Figure 2) shows the Cypriots' teachers' beliefs about mathematical error. In specific, the diagram consists of three similarities groups, which are not connected to each other. This is an indication that Cypriot secondary school teachers do not have an overview about the concept of the error in mathematics, but they isolate some of its aspects.

At a first glance, we observe that the most significant similarity pair is found in the third similarity group and it is formed by the variables R16 and R17 (0.60). These variables attribute mathematical error to previous correct knowledge which is not appropriate in a new situation or to a confusion of the model needed for completing a task with an already known model. In the same group, strong similarity relationship is observed between the variables R13 and R19 (0.58). According to these, the errors are associated with inappropriate ways of teaching or with an inappropriate question for the students' ability. Therefore, this pair of variables indicates that the students' errors in math are attributed to the teacher, and especially to their teaching methods and the questions he/she uses that are not appropriate for the students' learning level. In general, all the variables that consist in this similarity group are linked with a significant, but very low similarity index (0.12). However, all these variables in this similarity group attribute the mathematical error to students who confuse their previous

knowledge with the new one and to teachers who don't adapt their teaching according to their students' needs.

As regards the other two similarity groups, we observe that they are formed by two pairs of variables each one. The strongest similarity relationship is found between the variables R8 and R9 (0.60). According to these, the errors are associated with lack of knowledge or with the text of the problem. The second similarity pair in this group consists of the variables R10 and R18 (0.54), according to which the errors are related to the way students are studying and preparing for mathematics or due to the fact that students are trying to meet their teachers' expectations without having awareness about them. This grouping indicates that the errors are due to the students. The variables grouped into the first similarity group show a very low similarity index (0.12) and attribute errors to mathematics mainly to the students, the lack of mathematical knowledge and the way they prepare themselves for an assessment.

The second similarity group consists of two pairs of variables that present a low, but significant similarity index (0.11). They are linked because they refer to variables related to the affective and cognitive domain of the students. However, the four variables are mainly linked due to the fact they refer to the factor "student", to which Cypriot teachers seem to attribute the mathematical error. In more details, we observe that the strongest similarity relationship is found between the variables R14 and R15 (0.59), which attribute the mathematical error to students due to their limited mathematical abilities and their incorrect or incomplete knowledge of a mathematical concept. The variables R11 and R12 also show a quite high similarity index (0.57). These variables attribute the mathematical errors to the students and especially to factors related to their affective domain, such as their attitudes towards mathematics and their psychological situation.

The similarity diagram below (Figure 3) shows the Swiss' teachers' beliefs about mathematical error. In this diagram, two similarities groups, which are not connected to each other, are formed. This is an indication that Swiss secondary school teachers of mathematics do not have an overall picture about the concept of the error in mathematics, but they isolate some of its aspects. At first glance, we notice that the strongest pair of similarity, in which a significant similarity index is observed, is found in the second subgroup in the first similarity group and it is formed by the variables R15 and R17 (0.79). According to these variables, the errors are due to the incorrect or

incomplete pre-existing knowledge for a mathematical concept or the confusion between the appropriate approach and the previous approach that is inappropriate for the specific case. Both variables seem to attribute the mathematical error to the student, who lags behind in the cognitive field. In the same subgroup we observe that there is a high similarity between the variables R9 and R18 (0.78), according to which mistakes are related either to the text and the formulation of mathematical problems or to the fact that students are trying to meet their teachers' expectations without have awareness about these. In addition, a low but significant similarity index (0.39) is observed between the two pairs of variables that form the second subgroup of variables. This clustering is due to the fact that most of the variables attribute the mathematical error to the student.

As regards the first subgroup of variables of the first similarity group, it also consists of two pairs of variables. The first pair is formed by the variables R8 and R14 (0.77), according to which the errors are related to the lack of mathematical knowledge and the limited mathematical abilities of the students. Therefore, this clustering is due to the fact that the two variables attribute the mathematical error to students because of their cognitive level. The second similarity pair in this subgroup consists of the variables R13 and R16 (0.77), according to which mistakes are related to inappropriate ways of teaching mathematics or to the pre-existing knowledge that are inappropriate for a new situation. The first statement attributes the mathematical error to the teacher while the second one to the student. However, the two pairs of variables that form the first subgroup of variables present a low similarity index (0.30) and they are grouped mainly because the most of these variables attribute the mathematical error to the student and his/her cognitive domain. In general, the two subgroups of variables are linked, presenting a very low, but significant similarity index (0.02). However, their grouping is due to the fact that the statements attribute the mathematical error mainly to the student.

As regards the second similarity group, it consists of two pairs of variables. In more details, the strongest similarity relationship is found between the variables R11 and R12 (0.79), which attribute the students' mathematical errors to factors related to their affective domain, such as their attitudes towards mathematics and their psychological situation. The second similarity pair in this group consists of the variables R10 and R19 (0.73), according to which the errors are related to the way that the students study and prepare themselves for math or to the inadequate question being asked in relation to students' abilities. The

first statement attributes the mathematical error to the student while the second one to the teacher. In general, the two pairs of variables that form the second similarity group show a low similarity index (0.34). However, this grouping emerges by the fact that the most of these variables attribute the mathematical error to the student and mainly to his/her affective domain.

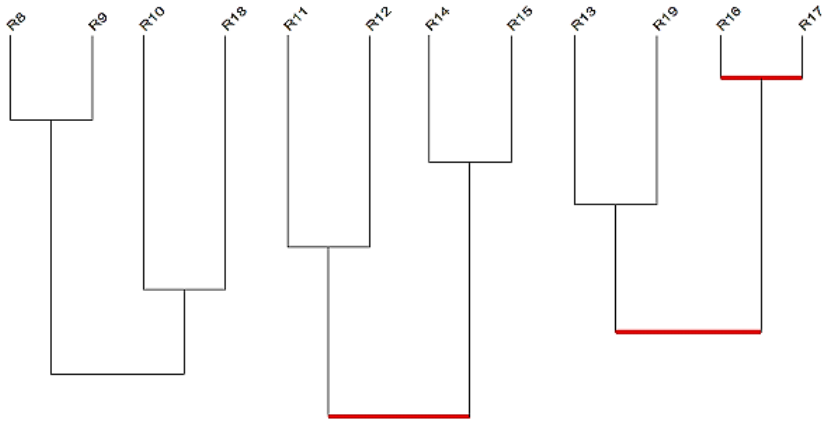

Figure 2: Similarity diagram on the Cypriot teachers' beliefs about errors in Mathematics

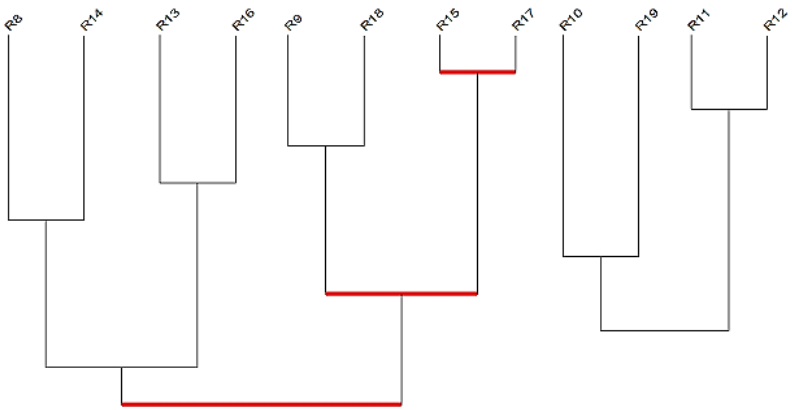

Figure 3: Similarity diagram on the Swiss teachers' beliefs about errors in Mathematics

4.3. A comparison between Cypriot and Italian teachers' and students' beliefs about errors in Mathematics

4.3.1. The concept of the mathematical error in Cypriot and Italian students

The Similarity Diagrams below (Figure 4) present the Cypriot and Italian students' beliefs about the concept of the mathematical error separately. In the diagrams we observe that two similarity groups are formed in each one, which are not linked. This is an indication that the Cypriot and Italian secondary school students don't have an overview of the concept of error in mathematics, but they isolate some of its aspects.

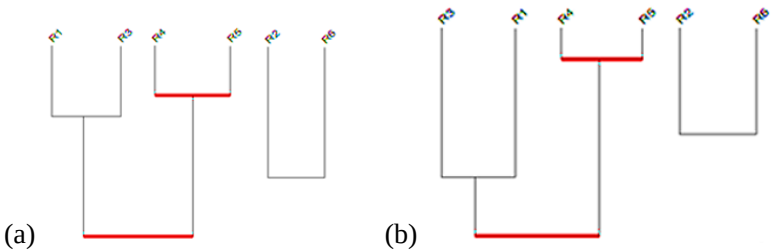

Figure 4: (a) Similarity diagram on the Cypriot students' beliefs about errors in Mathematics and (b) Similarity diagram on the Italian students' beliefs about errors in Mathematics

At a first glance, we observe that the two similarity diagrams are identical. In specific, we notice that strongest similarity pair is formed by the variables R4 and R5, which appear significant similarity index. These variables are lying in the first similarity group and they are related with the teacher's role in the handling of the student's errors in mathematics. Strong similarity index is also observed between the variables R1 and R3, which refer to the understanding of the mathematical concept, which gained through the correction of errors. The above two pairs of variables compose the first similarity group, which is formed by variables related to the students' cognitive domain and present significant similarity index between them.

The second similarity group includes only one pair of variables, which refer to the affective domain of the students. These variables are

the R2 and R6, which present similarity index equal to 0.63 and 0.66, respectively.

4.3.2. The concept of the mathematical error in Cypriot and Italian teachers

The following similarity diagram (Figure 5) shows the Cypriots' teachers' beliefs about mathematical error. In specific, the diagram consists of three similarities groups, which are not connected to each other. This is an indication that Cypriot secondary school mathematicians do not have an overall picture about the concept of the error in mathematics, but they isolate some of its aspects.

At a first glance, we observe that the most significant similarity pair is found in the third similarity group and it is formed by the variables R16 and R17 (0.60). These variables attribute mathematical error to previous correct knowledge which is not appropriate in a new situation or to a confusion of the model needed for completing a task with an already known model. In the same group, strong similarity relationship is observed between the variables R13 and R19 (0.58). According to these, the errors are associated with inappropriate ways of teaching or with an inappropriate question for the students' ability. In general, all the variables that consist in this similarity group attribute the mathematical error to students who confuse their previous knowledge with the new one and to teachers who don't adapt their teaching according to their students' needs.

As regards the other two similarity groups, we observe that they are formed by two pairs of variables each one. The strongest similarity relationship is found between the variables R8 and R9 (0.60). According to these, the errors are associated with lack of knowledge or with the text of the problem. The variables grouped into the first similarity group show a very low similarity index (0.12) and attribute errors to mathematics mainly to the students. The second similarity group consists of two pairs of variables that present a low but significant similarity index (0.11). They are linked because they refer to variables related to the affective and cognitive domain of the students. However, the four variables are mainly linked due to the fact they refer to the factor "student", to which Cypriot teachers seem to attribute the mathematical error.

The similarity diagram below (Figure 6) shows the Italians' teachers' beliefs about mathematical error. In this diagram, three similarities groups, which are not connected to each other, are formed. This is an indication that Italian secondary school teachers of mathematics do not have an overall picture about the concept of the error in mathematics, but they isolate some of its aspects.

At first glance, we notice that the strongest pair of similarity, in which a significant similarity index is observed, is found in the third similarity group and it is formed by the variables R11 and R18 (0.57). These variables are grouped as referring to statements that attribute mathematical error to students, mainly due to emotional factors. In the same similarity group, there is also a strong similarity relationship between the R15 and R16 variables (0.55), according to which the errors are related to previous pre-existing knowledge, which are either incorrect or inappropriate in a new situation. Generally, the variables that form the last similarity group present a significant, but very low similarity index (0.10) and seem to attribute the mathematical error to students for both emotional and cognitive factors.

Regarding the other two similarity groups, we observe that they consist of two pairs of variables each one. More specifically, the strongest similarity relationship in the first similarity group is found between the R8 and R14 variables (0.54), according to which the errors are related to the lack of mathematical knowledge and the limited mathematical skills of the students. Therefore, this clustering occurs due to the fact that the two variables attribute the mathematical error to students because of their cognitive level. The second similarity pair in this group consists of the variables R10 and R17 (0.50), according to which the errors are associated with the way the student studies and prepares himself/herself or to the confusion between the appropriate approach and the previous approach which is inappropriate in the particular situation. Once again, we notice that the grouping of variables in the first similarity group presents a very low similarity index (0.07); however, it results from the fact that all variables attribute the mathematical error to the student.

The second similarity group consists of two pairs of variables, which are connected with a low similarity index (0.09). They are linked because they mainly represent variables that attribute the students' mathematical error to the teacher. Analytically, we observe that the strongest similarity relationship is found between the variables R9 and R19 (0.56), which attribute the mathematical error the formulation of the text of the problem and the inappropriate question, in relation with students'

abilities. The variables R12 and R13 present a relatively high similarity index (0.53). According to these variables, the errors are associated with the psychological situation of the students, but also with inappropriate ways of teaching mathematics.

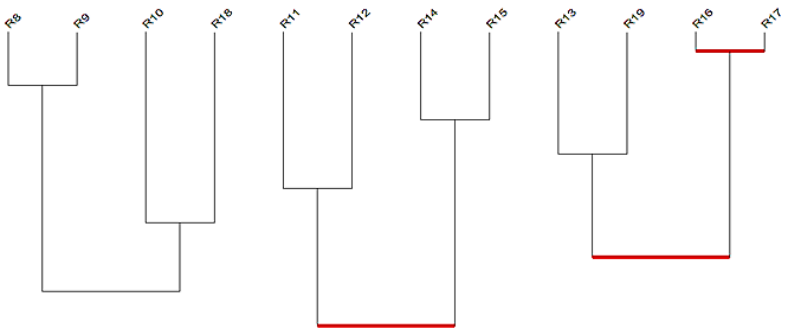

Figure 5: Similarity diagram on the Cypriot teachers' beliefs about errors in Mathematics

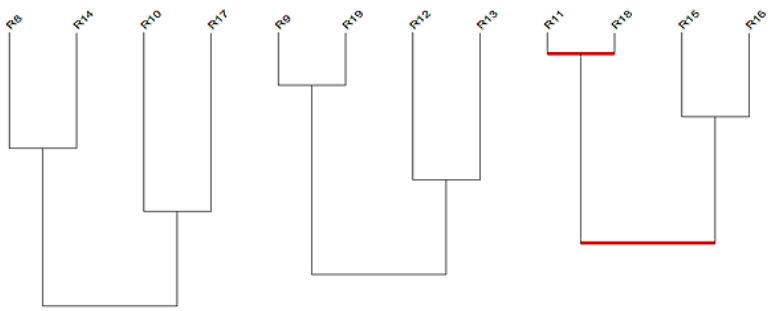

Figure 6: Similarity diagram on the Italian teachers' beliefs about errors in Mathematics

5. Discussion – Conclusions

Based on the presentation in the literature review of some of the different definitions and some of the different methods and techniques of the formative assessment we could assure that formative assessment is a multidimensional "notion". This "notion" is part of the teaching-learning process and regulates it. It is supposed to identify in an analytical way, the strengths and weaknesses of student's learning, in order to allow teachers to reflect on and modify their own practices. In Mathematics, the teachers could identify their students' weaknesses principally by reflection on their errors. That is way we have focused in the present paper in a detailed presentation of students' and teachers' beliefs about errors in three different countries.

By comparing the results of the two similarity diagrams illustrating the students' beliefs in Cyprus, Italy and Switzerland, about the errors in Mathematics, we observe that despite the possible differences between the pedagogical approaches of the teachers in the three countries in question and the curriculum content of mathematics, there is a convergence between the beliefs of the students. This is explicit from the corresponding similarity diagrams which are very similar.

Accordingly, by comparing the results of the similarity diagrams illustrating teachers' beliefs about the errors in Mathematics, we observe that there is no agreement between the Cypriot, Italian and Swiss teachers. This inconsistency between the teachers' beliefs is probably due different factors such as: the different educational system of each country, the different studies of the teachers, the characteristics of their personality et al.

In general, we observe that there is no agreement between the teachers' beliefs about errors in Mathematics, in the different countries. On the contrary, it seems that a universal dimension of the concept of error in mathematics on the part of students does exist. This dimension is stable regardless of the differences in culture and education between different countries. Thus, the dimension of using errors as source for feedback and feedforward actions must not be neglected in the teaching process. In relation to this, Cauley and McMillan (2010) explain that by showing the students specific misunderstandings or errors that frequently occur in a content area or a skill set, and showing them how they can adjust their approach to the task, students can see what they need to do to maximize their performance.

In addition, more attention must be given on assessing students through ways that allow an interaction between teachers and students and provide more chances for understanding the students' cognitive processes, their knowledge, misconceptions and strategies. Such knowledge is powerful because students have a good understanding of what they are doing and why the teacher provides them feedback and these help them understand what they are learning, to set goals, and to self-assess (Cauley & McMillan, 2010).

Nevertheless, if we consider the different nature of errors in mathematics, errors linked to epistemological or didactic obstacles, errors linked to the didactic contract or others (Brousseau, 1997), we conclude that the same type of research must be repeated by proposing a greater variety of statements to students. Thus, one could probably validate an implicit model of students' beliefs about the concept of mathematical error in secondary education. Thereafter, gaining access to the students', but also the teachers' beliefs will give us the opportunity to design relevant teaching material, based on their needs, in order to have the chance to achieve a change in classroom practices towards the effective implementation of formative assessment.

REFERENCES

ASCH, R. L. (1976) Teaching beliefs and evaluation. *Art Education, 29*(6), 18-22.

BLACK, P., & WILIAM, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139-148.

BLACK, P., & WILIAM, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability, 21*(1), 5-31.

BROUSSEAU, G., 1997. *Theory of didactical situations in mathematics 1970-1990*. Dordrecht: Kluwer Academic Publishers.

BROWN, G.T.L. (2004). Teachers' conceptions of assessment: Implications for policy and professional development. *Assessment in Education: Principles, Policy and Practice, 11*(3), 301-318.

CALDERHEAD, J. (1996) Teachers' beliefs and knowledge, in: D. C. Berliner & R. C. Calfee (Eds) *Handbook of educational psychology* (New York, Simon & Schuster Macmillan), 709-725.

CAULEY, K. M., & MCMILLAN, J. H. (2010). Formative assessment techniques to support student motivation and achievement. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 83*(1), 1-6.

CHERUBINI, G. (2002). Gli insegnanti e l'apprendimento. *Scuola e Città, 1,* 69-80.

CHRISTODOULOU, T., GAGATSIS, A, & ANASTASIADOU, S. (2017). Enquête Internationale Comparative: Les Croyances Et Les Attitudes Des Élèves Du Secondaire Inférieur Relatives Aux Erreurs En Mathématiques. In Régnier, J. C., Gras, R., Couturier, R., & Bodin, A. (Eds.), *Analyse Statistique Implicative* (pp. 402-415). Université Bourgogne Franche-Comté –Besançon: icar.

CLARIANA, R.B. (1990). A comparison of answer-until correct feedback and knowledge-of-correct-response feedback under two conditions of contextualization. *Journal of Computer-Based Instruction 17* (4), 125-129.

CLARK, I., (2011a). Formative Assessment: Policy, Perspectives and Practice. *Florida Journal of Educational Administration and Policy, 4* (2), 158-180.

CLARK, I., (2011b).Formative assessment and motivation: *Theories and themes. Prime Research on Education (PRE), 1*(2), 27-36.

CLARKE, D. J., CLARKE, D. M., & Lovitt, C. J. (1990). Changes in mathematics teaching call for assessment alternatives. IN T. Cooney & C. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s.* Reston, VA: National Council of Teachers of MATHEMATICS.

CROOKS, T. J. (1988). The impact of classroom evaluation practices on students. *Review of educational research, 58*(4), 438-481.

DE CORTE, E., OP'T EYNDE, P., VERSCHAFFEL, L. (2002). Knowing what to believe: The relevance of students' mathematical beliefs for mathematics education. In B.K. Hofer

& P.R. Pintrich (Eds.), *Personal epistemology. The psychology of beliefs about knowledge and knowing* (pp. 297–320). Mahwah, NJ: Lawrence Erlbaum Associates.

DESFORGES, C. (1989). *Testing and Assessment.* London: Cassell.

ECONOMOU, P. (1995). How teachers of mathematics confront students' errors. In G. Philippou, C. Christou, & A. Kakas (Eds.), *Proceedings of the Second Panhellenic Conference on Mathematics Education and the Informatics in Education,* (pp. 383– 400). Nicosia: Sighroni Epoxi.

FRANKE, M.L., FENNEMA, E., & CARPENTER, T.P. (1997). Teachers creating change. Examining evolving beliefs and classroom practice. In E. Fennema & B.S. Nelson (Eds.), *Mathematics teacher in transition. The studies in mathematical thinking and learning series* (pp. 225–282). Mahwah, NJ: Lawrence Erlbaum Associates.

GAGATSIS, A. & KYRIAKIDES, L. (2000). Teachers' attitudes towards their pupils' mathematical errors. *Educational Research and Evaluation, 6*(1), 24–58.

GAGATSIS, A., & CHRISTOU, C. (1997). Errors in mathematics: A multidimensional approach. *Scientia Paedagogica Experimentalis, 34*(1) 89–116

GAO, W., Du, Y.H., & YU, X. (2006). Higher education examination system reform and the nurturing of innovative abilities. *Journal of Inner Mongolia Normal University, 19*(5), 433-445.

GAROFALO, J. (1989). Beliefs and their influence on mathematical performance. *Mathematics Teacher, 82,* 502–505.

CHARNAY, R. (1989). *Les enseignements des mathématiques et les erreurs de leurs élèves*. Grand N. Institute de Recherche sur l'Enseignement de Mathématiques (IREM) de Grenoble.

GRAS R., SUZUKI E., GUILLET F. & SPAGNOLO F. (Eds) (2008). *Statistical Implicative Analysis.* Springer-Verlag, Berlin-Heidelberg.

HARLEN, W. (2000). *Teaching, learning and assessing science* (3rd ed.). London: Paul Chapman Publishing.

KARDASH, C.M., & HOWELL, K.L. (2000). Effects of epistemological beliefs and topic-specific beliefs on undergraduates' cognitive and strategic processing of dual-positional text. *Journal of Educational Psychology, 92*, 524–535.

KLUGER, A.N. & DENISI, A. (1996). The effects of feedback interventions on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin 119* (2), 254-284.

KYRIAKIDES, L. & CAMPBELL, R.J. (1999). Primary teachers' perceptions of baseline assessment in mathematics. *Studies in Educational Evaluation, 25*, 109-130.

KYRIAKIDES, L. (1999). Research on baseline assessment in mathematics at school entry. *Assessment in Education: Principles, Policy and Practice, 6*(3) 357-375.

LEE, C., & WILIAM, D. (2005). Studying changes in the practice of two teachers developing assessment for learning. *International Journal of Teacher Development, 9*(2), 265-283.

MASON, L. (2000). Role of anomalous data and epistemological beliefs in middle students' theory change on two controversial topics. *European Journal of Psychology of Education, 15*, 329–346.

MELMER, R., BURMASTER, E., & JAMES, T. K. (2008). *Attributes of effective formative assessment.* Washington, DC: Council of Chief State School Officers. Retrieved from http://www.ccsso.org/publications/details.cfm?PublicationID=362

MICHAEL–CHRYSANTHOU, P., GAGATSIS, A. & VANNINI, I. (2014). Formative assessment in mathematics: a theoretical model. *Acta Didactica Universitatis Comenianae – Mathematics, 14*, 43-70.

MICHAEL-CHRYSANTHOU, P. & GAGATSIS, A. (2015). Students' beliefs for formative assessment in mathematics teaching and learning. *EAPRIL Conference Proceedings 2014, Issue 1* (pp. 178-193). ISSN 2406-4653.

MILHAUD, H. (1980). *Le comportement des maitres face aux erreus des eleves,* [Teachers' attitudes towards pupils errors]. DEA

[Masters thesis] de Didactique des Mathematiques, IREM de Bordeaux.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (1995). *Assessment standards for school mathematics.* Reston, VA: NCTM.

NICOL, D., & MACFARLANE-DICK, D. (2004). Rethinking formative assessment in HE: a theoretical model and seven principles of good feedback practice. In C. Juwah, D. Macfarlane-Dick, B. Matthew, D. Nicol, D. & Smith, B. (Eds.), *Enhancing student learning though effective formative feedback.* York: The Higher Education Academy.

PALINSCAR, A., MAGNUSSON, S., MARANO, N., FORD, D., BROWN, N. (1998). Design principles informing and emerging from the GisML Community. *Teaching and Teacher Education, 14* (1), 5-19.

PEHKONEN, E. (1998). Teachers' conceptions on Mathematics Teaching. In M. Hannula (Ed.), *Current state of research on mathematical beliefs V: Proceedings of the MAVI-5 workshop,* August 22–25, 1997 (pp. 58–65). Helsinki, Finland: Department of Teacher Education, University of Helsinki.

POPHAM, W. J. (2008). *Transformative assessment.* VA: ASCD.

RAMSDEN, P. (1997). The context of learning in academic departments. *In The experience of learning: Implications for teaching and studying in higher education,* 2nd ed., ed. F. Marton, D. Hounsell and N. Entwistle, 198–217. Edinburgh: Scottish Academic Press.

SADLER, D. R. (1998). Formative assessment: Revisiting the territory. *Assessment in education, 5*(1), 77-84.

SCHOENFELD, A.H. (1983). Beyond the purely cognitive: Beliefs system, social cognition, and metacognition as driving forces in intellectual performance. *Cognitive Science, 7*, 329–363.

SCHRAW, G., DUNKLE, M.E., & BENDIXEN, L.D. (1995). Cognitive processes in well-defined and ill-defined problem solving. *Applied Cognitive Psychology, 9*, 523–538.

STRUYVEN, K., DOCHY, F. & JANSSENS, S. (2005). Students' perceptions about evaluation and assessment in higher education: A review. *Assessment and Evaluation in Higher Education, 30*(4), 325–341.

THOMPSON, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.*), Handbook of research on mathematics leaching and learning* (pp. 127-146). New York: Macmillan.

VAN DE WALLE, A. J., KARP, S. K., & BAY-WILLIAMS, M. J. (2013). *Elementary and Middle School Mathematics: Teaching Developmentally* (8 ed.). United States of America: Pearson.

VANNINI, I. (2009). *La Qualità nella didattica*. Trento: Erickson.

VANNINI, I. (2012). Come cambia la cultura degli insegnanti. *Metodi per la ricerca empirica in educazione.* Milano: Franco Angeli.

Appendix. The statements of the research instrument of the research

Express your opinion about the following statements, by circling the proper number in the scale (from 1=never to 4=always).	
R1	Correcting my mistakes helps me to understand better a mathematical concept.
R2	My mistakes in math discourage me.
R3	After an assessment in math, my teacher wants to verify if I have understood the mistakes that I have made.
R4	My teacher uses our mistakes and interests to plan the next mathematics lesson.
R5	My math teacher wants to be with me while I am correcting my mistakes.
R6	If I make mistakes in math I deserve a low grade.

Table 1: Statements about the use of error in Mathematics from the students' questionnaire

Express your opinion about the following statements, by circling the proper number in the scale (from 1=strongly disagree to 4= strongly agree).	
R8	Errors are associated with lack of knowledge.
R9	Errors are associated with the text of the problem.
R10	Errors are associated with the way the student studies and prepares himself/herself.
R11	Errors are associated with student's attitude towards mathematics.
R12	Errors are associated with the psychological situation of the student.
R13	Errors are associated with inappropriate ways of teaching.
R14	Errors are due to the limited capabilities of students.
R15	Errors are due to wrong or incomplete knowledge about a concept taught previously.
R16	Errors are due to previous correct knowledge which is not appropriate in a new situation.
R17	Errors are due to a confusion of the model needed for completing a task with an already known model.
R18	Errors are due to the students' tendency to fulfill their teacher's wishes without examining them.
R19	Errors are due to the fact that an inappropriate question for the ability of the student is given.

Table 2: Indicative statements about the use of error in Mathematics from the teachers' questionnaire

Abstract

The present paper discusses lower secondary school teachers' and students' beliefs about Formative Assessment in mathematics in Cyprus, Switzerland and Italy. Based on teachers' and students' beliefs related to formative assessment, this research is an attempt to compare the teachers' and students' beliefs in the different countries under study, regarding to the handling of the error in mathematics classroom. In total, 192 teachers and 1108 lower secondary school students participated in the survey and they completed a questionnaire about the formative assessment. The findings of the study reveal that the students' beliefs in the three countries are identical, whereas teachers' beliefs about mathematical error differ enough in the particular countries.

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