PRACTICES OF ITALIAN TEACHERS WITH THE DERIVATIVE CONCEPT: A PROBLEMATIC MEETING BETWEEN ALGEBRA AND ANALYSIS IN SECONDARY SCHOOL

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This paper reports on a study from a wider project of thesis, whose main focus is investigating, in the secondary school context, the transition between two important mathematics domains Algebra and Analysis. Following Kuhn (1969), we detect some fundamental paradigm shifts between the two. They represent a change in the way of thinking about and working with functions, so that the related MWSs come to be differently structured. The paper focuses on a fundamental process in mathematics, based on the idea of "generic". Through the analysis of how a teacher manages her students' work on the derivative function, our aim is discussing how the Analysis MWS differs from the Algebra MWS, in terms of cognitive processes and especially with respect to their genesis.

INTRODUCTION

In the Italian secondary school, Algebra and Analysis are introduced in two different and consecutive moments. Algebra is mainly presented as literal calculus and solution of first grade equations at the end of lower secondary school (grade 8) and acquires an increasing importance during the first two years of upper secondary school (grade 9-10), where it is identified with the study of numeric systems and of their properties, and the resolution of second degree equations, inequalities and systems. Analysis, instead, prevails as elementary Calculus in the curricula of the last year of upper secondary school (grade 13) with the systematic study of real functions of a real variable, limits, differential and integral calculus. Between these two phases, a preliminary work on elementary functions occurs, whose features are mainly algebraic and graphical. In the secondary school, the approach to Calculus is strongly based on the algebraic work, but is this enough to make students understand suitably the fundamental concepts of Calculus? Is a student well-prepared to face the study of Analysis at the university? Grounded on such concerns, we developed the idea of investigating what kind of articulation exists between the two mathematical domains.

THEORETHICAL FRAMEWORK

From an historical point of view, Analysis has been introduced in mathematics as infinitesimal calculus. Leibniz, Newton and the contemporary mathematicians began to notice some anomalies in the algebraic calculus when it involved infinitely small quantities. Such anomalies could not be explained within the universally accepted paradigms. As it is well known, the concept of paradigm is developed by Kuhn in his work on the scientific revolutions (Kuhn, 1969), as the set of beliefs, techniques and principles shared by a scientific community (Kuzniak, 2011). We could affirm that the introduction of the “infinitely small” has caused a paradigm shift in mathematics. It has entailed the adoption of a local perspective on functions and a new definition of equality of functions. Thus, two large reference Mathematical Working Spaces (MWSs from now on, Kuzniak 2011) are identifiable, that of Algebra and that of Analysis. One of the distinctions between the two MWSs relies on this shift of paradigm.

In this paper, we consider the notion of generic that is a fundamental idea in mathematics. It is at the base of a process we call “genericization”, that is generation of the generic case. We find this idea already in Euclid (see his “generic” proof of the infinity of prime numbers) and in some speculations of philosophers (e.g. Locke: see below), but it was particularly exploited within algebraic geometry at the turning of 19^{th} century. The geometers introduced explicitly the notion of “generic point”, and the related practices in their discipline. However, while generic points were widely used, it is not easy to find their definition.

For that we can refer to Van der Waerden (1926):

> Indeed, by generic point of a variety, one usually means, even if this is not always clearly explained, a point which satisfies no special equation, except those equations which are met at every point (p. 197)

and to Enriques & Chisini (1915):

> The notion of a generic ‘point’ or ‘element’ of a variety, i.e., the distinction between properties that pertain in general to the points of a variety and properties that only pertain to exceptional points, now takes on a precise meaning for all algebraic varieties. A property is said to pertain in general to the points of a variety V_n, of dimension n, if the points of V_n not satisfying it form – inside V_n – a variety of less than n dimensions (p. 139).

An interesting point of view is given by Speranza (1996), who analysed the idea of “general triangle” through the words of the great philosopher Locke (1690): “[The general triangle] must be neither oblique nor rectangle, neither equilateral, equicrural, nor scalenon: but all and none of these at once”[1] (in Speranza, 1996, p.15). The idea of generic occurs in the mathematical practices leading to the research of a “generic stereotype” which represents all the basic features desired without any added specific singularity.

In Analysis, for example, the process of genericization occurs when we know how a function behaves for some values of x and we shift our reasoning on a “generic abscissa x”, which must belong to the function domain, but has no particular added characteristic.

Genericization is a practice used also in Algebra with the work on generic examples. For instance, let us imagine that we have to decide if the sum of two even numbers is even or odd. In Algebra, we can proceed empirically, testing several cases (e.g. 2 + 4 = 6, 4 + 8 = 12, and so on) and, then, inducing the general property: the sum is an even number. This is an example of generalization, that means inducing the general case, from a sequence of particular cases. From an epistemological point of view, generalization is an empiric induction, which entails an empiric, but not real proof.

However, we can follow another way in order to decide if the sum of two even numbers is even or odd: we can reason on a particular example, giving emphasis to a general feature that characterizes all the examples similar to the proposed one.

$$\begin{array}{r} 14 = 7 + 7 \\ 22 = 11 + 11 \\ \hline 36 = 18 + 18 \end{array}$$

As a consequence, it becomes useless to provide other examples: the given one can be conceived as generic. This is a "generic proof", that is "a proof carried out on a generic example" (Leron & Zaslavsky, 2009).

Inducing the general case and reasoning on the generic case are two distinct processes in mathematics. The former is typical of the transition from Arithmetic to Algebra, the latter instead is common to different mathematical domains. We saw examples from the literature within Geometry and Algebra domains. The purpose of this study is to investigate the idea of generic and the associated process of genericization also in Analysis.

In the context of the Symposium, this paper aims to give a specific contribution to the chosen topic, discussing how the Analysis MWS differs from the Algebra MWS, in terms of cognitive processes and especially with respect to their genesis.

Since a mathematical working space is structured on two levels, the epistemological and the cognitive planes, we need firstly to precise on which epistemological basis the process of genericization is generated. This process is often related to the use of the universal quantifier "∀": the mathematical sign for "for each", "for every" and "for all". We can remark that, on the one hand, the natural language makes a distinction between the terms "every" and "all". They can both be used to talk about things in general, but "every" has the distributive meaning of "all in each of its parts". So, for example, if we say "Every Italian citizen has a name and a surname" we want to underline the fact that "if taken one by one", to all Italian citizens has been given a name and a surname. But we would say "All Italian citizens older than 18 has the right to vote" in order to stress that the totality of Italian people can vote, without inner distinctions. In mathematics, instead, the expressions "for every" and "for all" are completely equivalent from a logical point of view.

These differences in terms of linguistic interpretation of the mathematical sign "∀" become very significant when we focus on the transition from Algebraic to Calculus practices and theories in the classroom. In order to analyse this transition, and take into account classroom practices and theories with functions, we use the notion of "perspectives" (Rogalski 2008, Vandebrouck 2011). A perspective on a given function f can be pointwise, global or local, according to the character of the properties of f that are taken into account. Thus, a perspective on f can be:

- *pointwise*, when one considers properties which depend only on the value of f in a particular point x_0 (for instance, "$f(x_0) = 3$");

- *global*, when one considers properties which are valid on intervals (for instance, "*f* is increasing in the interval [*a*,*b*]");
- *local*, when one considers properties which depend on the values of *f* in a neighbourhood of a point (for instance, "*f* is continuous in x_0").

Following Vandebrouck (2011), we remark that some global properties actually are universal pointwise properties: that means pointwise properties verified point by point, for every[2] point of the interval. So, we can identify another perspective, that we call *universal pointwise* perspective. All the universal pointwise properties are global, but the vice versa does not always occur. However, in the case of global properties that cannot be defined in a universal pointwise way, Vandebrouck observes that "global perspective and quantification become fundamental" (Vandebrouck, 2011, p.157). Thus, introducing global properties only as universal pointwise may not help the students to properly grasp a global perspective on the involved function. This means that knowing that some properties are valid point by point, for each point of a given interval (universal pointwise perspective), might not imply a full global perspective on the considered function. In order to account for this assertion, an example will be shown below in our analysis.

Through the lens of perspectives, we can better describe the general work done on functions in Algebra and Calculus domains. Within the Algebra domain, at least in Italy, it may not be explicit that the work is on functions, although every given literal expression could be the analytic expression of a certain function. Thus, one deals with a universal pointwise perspective, since the properties or the algebraic expressions are conceived valid for each *x* of the domain of definition. The work in Calculus domain involves pointwise and global perspectives on functions, and for the first time within the teaching of functions, also local implications are considered. We will illustrate below how perspectives can help clarifying that the process of genericization can be seen as a shift from a pointwise to a universal pointwise perspective on the involved function.

In our research, two important indicators of the way of thinking about an object, and so of the perspective adopted on it, are the praxeologies developed with a particular type of task involving the object and the semiotic resources activated in the solution.

Praxeologies are a basic construct in the *Thèorie Anthropologique du Didactique* by Chevallard (Chevallard, 1992): they describe the set of practices used for solving a certain type of task. A praxeology is composed of four elements: a type of task, a technique to solve it, a technology that is a justification of the used technique and a theory in which the technology finds justification.

Moreover, an implicit perspective can be revealed also by the semiotic resources that one activates for facing the posed problem, for finding a technique to solve it, for justifying or rejecting processes of reasoning and for showing theoretical properties of the object in question. Within the theory of the *semiotic bundle* (Arzarello, 2006), different semiotic resources, such as speech, gestures, written signs (drawings, sketches, symbols, …), are simultaneously and interactively activated while doing mathematics.

RESEARCH OBJECTIVE AND METHODOLOGY

As said before, the aim of this paper is to describe and analyse the process of genericization in Analysis, and our research approach is based on the idea of generic. In particular, we want to study the practice and the acquisition of the process of genericization in a secondary school context, so within the study of Calculus. Our focus is on the teacher's role in this dynamics and the question we ask is *How do teachers manage this process with their students?*[3] In this sense, the present study is linked to one of the aims, which Kuzniak (2011) highlights as fundamental for systematically exploring and describing a mathematical working space: "[Ces outils doivent permettre] la description des enjeux épistémologiques et didactiques propres à chaque domaine mathématique en relation avec une approche par paradigmes" (Kuzniak, 2011, p.22).

We chose the specific topic of the derivative and we observed the practices of three Italian teachers. We interviewed each teacher, and then we video-recorded during their lessons with 13th grade students (18-19 years old, last year of secondary school). In our analysis, we are interested in the types of praxeologies the teachers build and develop in class and the types of semiotic resources that are used. Indeed, this could certainly have effects on the perspectives adopted by students on a function and its derivatives.

By following Chevallard's "method of didactic moments" (Chevallard, 1999) to analyse teachers' didactic praxeologies, our focus is on three of his six didactic moments. More precisely, we are interested in the moment of the first significant meeting with a particular type of task, the moment of exploration of the type of task and of construction of a technique for it, and finally the technological-theoretical moment. We chose to analyse the teacher facing a specific type of task triggering the idea of generic: algebraically representing the derivative function. Every teacher finds herself to deal with this type of task during the lessons and they have all just defined the derivative of a function f in a point x_0, so they all start from a pointwise perspective on f'.

The analysis of the teachers' practices is based on perspectives in terms of both praxeology components and semiotic resources. More precisely, we are interested in

- the pointwise, global or local character of the given type of task, of the employed technique and of the proposed justification, as well as the articulation between them;
- the role of the chosen semiotic resources in highlighting the pointwise, global or local perspective on the involved functions.

TWO EXAMPLES OF TEACHERS' PRACTICES WITH THE IDEA OF GENERIC

The case of teacher T1

T1 has just introduced the derivative of a function *f* in a point x_0 as the limit of the incremental ratio of *f* as the increment *h* goes to 0 (Fig. 1), starting from the problem of the tangent.

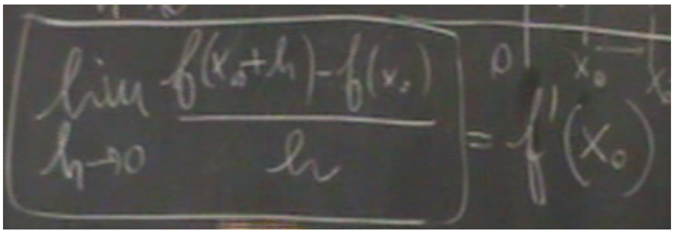

Figure 1: T1's technique for the derivative of a function *f* in a point x_0.

The teacher proposes to work on an example, in order to practice the new technique. She gives and solves the following task: determining the derivative of the function $y=x^2$ in the point of abscissa $x_0=2$. She uses the technique in Fig. 1 and finally finds $f'(2)=4$. Notice that the given task and the technique used for solving it are pointwise on the function f'. Here is how she comments the exercise and introduces the following one (in the transcript, Sn stands for the student n).

1 T1: Now, summarizing, what have we done? The concept of derivative, but calculated in a point. (*She points to an imaginary point in front of her, with her left hand, Fig. 2 on the left*) [...] The derivative of a function in a point, what does it give?

2 S1: A coefficient.

3 T1: A number, exactly. But now let's make a step forward. We have $y = x^2$ and we have calculated the angular coefficient[4] of the tangent line in the point $x_0 = 2$. (*She repeats the same previous gesture with her left hand, as in Fig. 2 on the left*) If I ask you now "What is the value of the angular coefficient in the point with abscissa $x_0 = 5$?" One should again work hard and do all the calculation. Right? In $x_0 = 1$... and so on. (*She turns her hands like something that unrolls*) You see, it's not so convenient, also from a practical point of view.

4 T1: So, what shall I do? The calculation in a generic point *x*. (*She joins the fingers of her right hand and then turns them down on the left palm which is open upwards, Fig. 2 on the right*) Ok? That is I call it *x*, instead of x_0. (*She repeats the same previous gesture with her right hand, as in Fig. 2 on the right*)

5 T1: And now we must be really careful! I call it *x*. Which outcome do I expect?

6 S2: A function.

7 T1: Can it be a number?

8 S3: With *x*.

9 T1: Yes, it will be a function of *x*. So, you understand that we can speak about "derivative function", which will be again a function of *x*.

10 S4: And then we can replace inside it...

11 T1: Perfect! S4 is saying “Of course, then, if I want the coefficient of the tangent in the point $x_0 = 5$, it will be sufficient to put $x = 5$ in the derivative function”. Let’s do it!

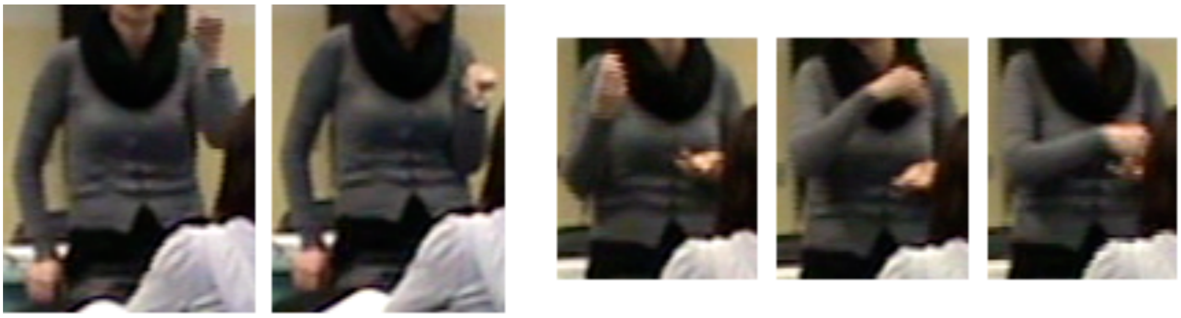

Figure 2: T1’s different gestures to indicate “the point x_0” (on the left) [lines 1 and 3] and the “generic point x” (on the right) [line 4].

After this comment, the teacher solves the same task following the same steps with x instead of x_0. She obtains $f'(x)=2x$.

The first utterance and the first gesture used by the teacher [line 1] stress that the starting perspective is pointwise on f'. Then, she underlines that making other numerical examples is actually useless, since every one of these examples ($x_0=5$, $x_0=1$, …) would always entail the same calculations done for $x_0=2$ [line 3]. The case of $x_0=2$ is becoming a generic example. T1 introduces a semiotic technique: the replacement of x_0 with x [line 4]. The previous argument [line 3] can be seen as a technology for this technique. More precisely, the limits and the non-convenience of a pointwise perspective on f' are the teacher’s justification for shifting to the generic sign x, which is universal pointwise. The generic sign x represents “every value of x_0”. Even T1’s gesture for accompanying the universal pointwise expression “the generic point x” [line 4] is different from the one used before for referring to “the point x_0” [line 1] (see Fig. 2). Finally, the teacher makes the students reflect upon the global expectations on the result [lines 5-9]: $f'(x)$ is expected to be globally a function of x, in which we can replace x with any number we wish [lines 10-11]. In this passage, another technique can be detected for finding out the derivative of f in a particular abscissa: the replacement of x with this abscissa in the expression of $f'(x)$. On the technological side, this technique is supported by the whole previous argument about the derivative function.

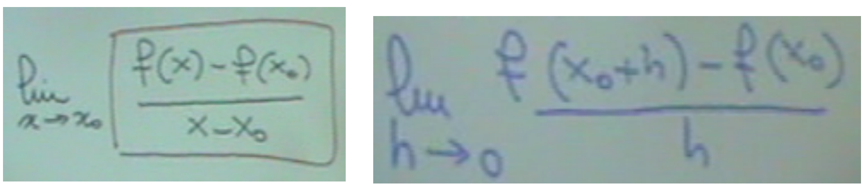

Figure 3: T2’s techniques for the derivative of a function f in a point x_0.

The case of teacher T2

During the first lesson, T2 and her students have found the formula of the derivative of a function f in a point x_0, starting from the gradient of the tangent

line to f in x_0. The result is the technique in Fig. 3 (on the left), that is the limit of the incremental ratio of f as x goes to x_0. At the beginning of the third lesson, T2 shows also the equivalent technique for h that goes to zero (Fig. 3 on the right).

Then, T2 gives a task to the students, by saying what follows (in the transcript, Sn marks the intervention of the student n).

1 T2: We are going to calculate the limit of the incremental ratio for the function $f(x) = x^2$. Try and write it on your own. So, try and calculate the derivative for x^2 in any point x_0 of it.

2 S1: Any point?

3 T2: Any point... As usual, let’s call it x_0.

Notice that the given task is universal pointwise on f' [“any point x_0”, line 1] and the techniques the students dispose of are pointwise on f' (Fig. 3). T2’s utterance in line 3 seems to reveal that the class is somehow familiar with the work on generic signs.

The students work alone for a while, the teacher walks through the classroom. When part of the class has solved the task, she makes all the steps at the whiteboard, obtaining $2x_0$. Then, she comments as follows.

4 T2: What have I discovered? I’ve discovered that when I have the function x^2, its derivative is... point by point... is $2x_0$.

5 T2: So, if I write a function here, and its derivative here (*she starts composing a table $f \mid f'$ at the blackboard*), I’ve discovered that the derivative of the function x^2 is $2x$. (*She writes “$x^2 \mid 2x$” in the first row of the table $f \mid f'$*) This is an automatic process, because if I have x^2, from this moment on, I won’t calculate the limit of the incremental ratio anymore. I know that its derivative is $2x$. I’ve calculated it once and for all, in the general case of any point x_0, so I have it.

The teacher’s first comment is universal pointwise [“point by point”, line 4], which actually recalls the character of the given task on f' [lines 1-3]. The sign x_0 is used in a generic sense to represent “any value x_0 of the abscissa”. Then, T2 suddenly jumps to a global conclusion [line 5], replacing x_0 with x. This semiotic technique is implicit in the change of variable from line 4 to line 5. T2 uses the table $f \mid f'$ as a resource to systematize. As for the perspectives however, x_0 is used as a universal pointwise sign, while x has a global meaning. From a technological point of view, at this stage T2 does not make explicit the shift from x_0 to x. It follows an opaque praxeology, whose technique and related technology are only hinted. A student intervenes about this variable change.

6 S2: The independent variable changes from f to f'... Is it x_0 or is it always the same?

7 T2: It is a point x. […] Let’s take $f(x) = x^2$, which I’m able to draw, that is the parabola (*she draws the curve*). What have we discovered and proved? That if I take any point x_0 (*she chooses a point x_0 on the x-axis*), then the angular coefficient of the tangent line in the point of abscissa x_0 [...] is $2x_0$.

So, if I draw the tangent line here (*she traces the tangent in the correspondent point on the parabola*), this straight line has $2x_0$ as angular coefficient (*she writes $m = 2x_0$*).

8 T2: What does it mean? It means that I can make x_0 vary as I want (*she moves her hand forwards and backwards, Fig. 5*)... At this point, I can write x instead of x_0, for convenience.

9 T2: And point by point I have a formula, that is the following (*she writes $f'(x) = 2x$*) which point by point (*she moves the stick as in Fig. 6*) tells me the value of the angular coefficient of the tangent line.

10 S2: $f'(x)$ gives me the angular coefficient...

11 T2: Yes, as x varies. So, the variable is the same. Point by point, here I have a function that point by point automatically, as a machine, tells me the angular coefficient of the tangent line.

12 S2: Only, I don't understand the passage... If we know that m is $2x$, $f(x)$ corresponds to y, while m corresponds to the tangent. How can they be equivalent? I don't understand.

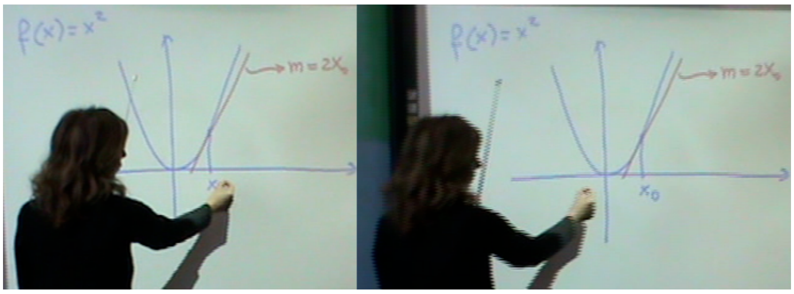

Figure 5: T2's gesture to accompany the words "I can make x_0 vary as I want" [line 8].

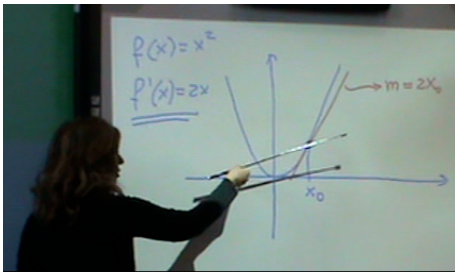

Figure 6: T2's gesture to show the tangent "point by point" [line 9].

As we can infer from S2's intervention [line 6], the opaque praxeology introduced by T2 induces doubts and confusion in the students. The teacher clarifies the generic role of x_0 and justifies, at a technological level, the shift from x_0 to x. Indeed, this change from a universal pointwise perspective to a global one was not so explicit in lines 4-5. T2 starts from stressing the pointwise basic character of the sign x_0, by choosing a particular point on the x-axis and the corresponding one on the parabola $y=x^2$. In order to make explicit the technology, she uses the graphical register as a resource. Then, she says "I can make x_0 vary as I want, so I can call it x" [line 8] and, at the same time, she moves her hand forwards and backwards on x-axis with a continuous gesture (Fig. 5). The previous hint to technology and technique [lines 4-5] is a little bit developed here, though in a very concise way. She justifies her change of x_0 into x (the used technique) as a convenience, since x_0 variation makes useless to specify any index for x. T2's utterance [line 8] and the continuous gesture (Fig. 5), combined together, underline the global aspect of the involved variables. This is the moment in which T2 moves from a universal pointwise perspective to a global one on the algebraic signs. But when she focuses again on the derivative function [line 9], she adopts a universal pointwise perspective ["point by point"] whereas the accompanying gesture (Fig. 6) is continuous and global on the variable x. Thus, we can remark that, in spite of a universal pointwise perspective made global on the sign x, the perspective on the derivative function remains universal pointwise. f' is presented "as a machine" which "point by point automatically tells me the angular coefficient of the tangent line" [line 11]. There is no explicit global reference to the function f'. This can probably explain the new doubt of the student S2 about the status of f' as a function [line 12]. He has a clear global image of the function f, thanks to the graphical register used in the teacher's drawing, but he can't understand how also the angular coefficient m (and so f') could behave like the function f does.

DISCUSSION AND IMPACT ON TEACHING

We have observed and analysed the practices of two teachers working on the derivative function. Notice that in both cases a semiotic genesis has occurred involving the signs x_0 and x in a process of genericization. T1 makes the numerical example become generic and reformulates the starting pointwise task in terms of a generic abscissa x. T2 immediately formulates the task in terms of a generic x_0 and then gives to it the global sense of x. Both praxeologies base their reasoning on a generic abscissa, denoted with x or x_0, in order to obtain a result that is valid as x varies globally.

This genericization on the signs x_0 and x is an algebraic practice that intervenes in the construction of a Calculus praxeology, whose object is the derivative function. How does this process on independent variables influence the work on the dependent variable expressed by $f'(x)$? For answering this question, we use the notion of perspectives. In the two analyzed examples, to the genericization on the independent variable corresponds a shift of perspectives on the derivative function: from pointwise to universal pointwise.

Now, let's recall our initial remark (see the paragraph on Theoretical framework) when we talked about the non-double implication between the universal pointwise perspective and the global perspective. Having constructed a universal pointwise perspective on f' does not automatically imply that this is also global. This passage has to be made explicit by the teacher. We can remark a difference in the work done by the two teachers on the derivative function. T1 explicitly discusses the global implications of the universal pointwise perspective on f': it is a function of the generic variable x [T1, line 9]. Instead, T2 gives a universal pointwise interpretation of the image $f'(x)$ for each point x: it is a formula which point by point gives us the angular coefficient of the tangent line [T2, line 11]. Without any global explicit indications, this work could inhibit the students from adopting a full global perspective on the derivative function.

From this discussion, three remarks turn to be relevant in teachers' Calculus practices. While working on functions, a teacher has to deal with old algebraic techniques and related technologies (such as the genericization), which intervene in the construction of the Calculus praxeologies. This can be a powerful bridge between the Algebra MWS and the Calculus one.

Thus, while the teacher structures the Analysis MWS in her classroom, she has to be very careful with regard to the perspectives activated on the involved functions. In particular, a delicate moment occurs when her goal is introducing the derivative function. Highlighting universal pointwise properties of f' in order to make the pointwise perspective evolve is a useful technique. But this may not be enough, at least not for all the students, for constructing a full global perspective on the derivative of a function as a function itself.

Finally, another remark can be on the semiotic resources used by the teacher to support the work in the classroom. Every semiotic resource, whether the user is conscious or not, has a certain potential with respect to a particular perspective. For example, drawing the graph of a function may foster a global perspective on it, and making a continuous gesture on the graph of a function (like T2's gestures in Fig. 5 and 6) may inhibit a pointwise perspective on it, in favor of a global one. But using a semiotic resource that fosters a global perspective may not be enough to obtain its activation, at least not for all the students. Therefore, in the work on functions, it is necessary not only a proper choice of the used resources, but possibly also the combined activation of other resources in order to focus the students' attention on the desired perspective.

In conclusion, perspectives and semiotic resources activated on functions, with attention to their mutual interactions, seem to be two important points in the teacher's management of the so-called *appropriate* Calculus MWS in class.

NOTES

1. This is the original quotation from the English translation we found in Speranza (1996, p.15).
2. "Every" is intentionally used here. We use "for each point" to stress the idea of the points taken one by one, but without a certain global image of the whole (universal pointwise, but not global). On the contrary, we use "for every point" to underline the idea of the whole in each of its parts (universal pointwise and also global).
3. This is a sub-question of the main research question of the theme 3: "This new topic deals with the role of the teachers and the interactions when forming a consistent but also efficient ETM. How to manage the interactions around the mathematical work in the classroom?"
4. In Italian, the coefficient of x in the equation $y = m\,x + q$ is called "coefficiente angolare", with reference to the property $m = \tan \alpha$, where α is the angle that the line forms with the positive direction of x-axis. Normally, an Italian teacher and her students refer to m as "coefficiente angolare" since the first time they study a straight line in the Cartesian plane (first year of upper secondary school). Although the slope of a line is strictly related to the angle which it forms with the x-axis, the name used partially hides this relation and students usually don't link, or at

least not directly, the *m*-value with the slope of the line. We think that the name used in Italian can evoke a certain mental image, different from the image linked to the English word "gradient" or the French word "coefficient directeur" for example, and we believe that this fact can actually influence the mathematical discussion and activity in classroom. For this reason, in the transcript we will use the literal translation "angular coefficient", instead of the correct English word "gradient".

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